Superfield Approach to Abelian 3-form gauge theory

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QFT 2011 (23 – 27 Feb. 2011) [IISER, Pune]
References


Superfield Approach: 

Bonora, Pasti & Tonin (81) 
Bonora & Tonin (82)

Ordinary fields of D-dimensional gauge theory

Superfields on (D, 2)-dimensional super manifold

\[ \phi(x) \longrightarrow \bar{\phi}(x, \theta, \bar{\theta}) : \text{Superfield} \]

(D) \[ x^\mu, \text{ (D, 2)-dimensions} \]

(i) \[ \theta^2 = \bar{\theta}^2 = 0, \quad \theta \bar{\theta} + \bar{\theta} \theta = 0 \]

(ii) \[ \partial_\theta^2 = \partial_{\bar{\theta}}^2 = 0, \quad \partial_\theta \partial_{\bar{\theta}} + \partial_{\bar{\theta}} \partial_\theta = 0 \]

Grassmannian Variables and derivatives
BRST approach to Gauge Theory

**BRST:** Becchi-Rouet-Stora-Tyutin

Local gauge symmetry
(First-class constraints)

BRST symmetry \((s_b)\)

Anti-BRST symmetry \((s_{ab})\)

\[ s_b^2 = 0, \quad s_{ab}^2 = 0 \]

Nilpotency property

\[ s_b s_{ab} + s_{ab} s_b = 0 \]

Absolute anticommutativity

\[ Q_b^2 = 0, \quad Q_{ab}^2 = 0, \quad Q_b Q_{ab} + Q_{ab} Q_b = 0 \]

- \(Q_b\): BRST charge
- \(Q_{ab}\): Anti-BRST charge
Plan of the Talk

• Why Abelian 3-form theory?
• Gauge field \( (B_{\mu\nu\eta}) \) and Ghost fields
• Horizontality condition \( \rightarrow \) symmetries
• Curci - Ferrari type restrictions
• Geometrical Aspects \( \rightarrow \) Gerbs
• Conclusions
Why Abelian 3-form theory??

D–Branes and their Physics

Gauge – Gravity Duality

Higher p–form (p = 2, 3, 4, …) Gauge Theories

(Super-)strings

Mathematics & Supersymmetric Field Theories

Non–commutative Field Theories

Higher Spin Gauge Theories
Abelian 3-form $B^{(3)}$ defines $B_{\mu\nu\eta}$ as

$$B^{(3)}(x) = \frac{1}{3!} (dx^\mu \wedge dx^\nu \wedge dx^\eta) B_{\mu\nu\eta}(x)$$

which can be generalized to super 3-form as

$$\tilde{B}^{(3)}(x, \theta, \bar{\theta}) = \frac{1}{3!} (dZ^M \wedge dZ^N \wedge dZ^K) \tilde{B}_{MNK}$$

$$\equiv \frac{1}{3!} (dx^\mu \wedge dx^\nu \wedge dx^\eta) \tilde{B}_{\mu\nu\eta} + \frac{1}{2} (dx^\mu \wedge dx^\nu \wedge d\theta) \tilde{B}_{\mu\nu\theta}$$

$$+ \frac{1}{2} (dx^\mu \wedge dx^\nu \wedge d\bar{\theta}) \tilde{B}_{\mu\nu\bar{\theta}} + \frac{1}{3!} (d\theta \wedge d\theta \wedge d\theta) \tilde{B}_{\theta\theta\theta}$$

$$+ \frac{1}{3!} (d\bar{\theta} \wedge d\bar{\theta} \wedge d\bar{\theta}) \tilde{B}_{\bar{\theta}\bar{\theta}\bar{\theta}} + (dx^\mu \wedge d\theta \wedge d\bar{\theta}) \tilde{B}_{\mu\theta\bar{\theta}}$$

$$+ \frac{1}{2} (dx^\mu \wedge d\theta \wedge d\theta) \tilde{B}_{\mu\theta\theta} + \frac{1}{2} (dx^\mu \wedge d\bar{\theta} \wedge d\bar{\theta}) \tilde{B}_{\mu\bar{\theta}\bar{\theta}}$$

$$+ \frac{1}{2} (d\theta \wedge d\bar{\theta} \wedge d\bar{\theta}) \tilde{B}_{\theta\theta\bar{\theta}} + \frac{1}{2} (d\theta \wedge d\theta \wedge d\bar{\theta}) \tilde{B}_{\theta\theta\bar{\theta}}$$
The above superfields provide hints for the existence of gauge fields and bosonic/fermionic (anti-) ghost fields of the theory.

**Identifications:**

\[
\begin{align*}
Z^M &= (x^\mu, \theta, \bar{\theta}), \\
\partial_M &= (\partial_\mu, \partial_\theta, \partial_{\bar{\theta}})
\end{align*}
\]

\[
\begin{align*}
\tilde{B}_{\mu\nu\eta} &= \tilde{B}_{\mu\nu\eta}(x, \theta, \bar{\theta}), \\
\tilde{B}_{\mu\nu\bar{\theta}} &= \tilde{F}_{\mu\nu}(x, \theta, \bar{\theta}), \\
\frac{1}{3!} \tilde{B}_{\theta\theta\theta} &= \tilde{F}_2(x, \theta, \bar{\theta}), \\
\frac{1}{2!} \tilde{B}_{\mu\theta\theta} &= \tilde{\beta}_\mu(x, \theta, \bar{\theta}), \\
\frac{1}{2!} \tilde{B}_{\theta\theta\bar{\theta}} &= \tilde{F}_1(x, \theta, \bar{\theta}), \\
\frac{1}{3!} \tilde{B}_{\bar{\theta}\bar{\theta}\bar{\theta}} &= \tilde{F}_2(x, \theta, \bar{\theta}), \\
\frac{1}{2!} \tilde{B}_{\mu\bar{\theta}\bar{\theta}} &= \tilde{\beta}_\mu(x, \theta, \bar{\theta}), \\
\frac{1}{2!} \tilde{B}_{\theta\theta\bar{\theta}} &= \tilde{F}_1(x, \theta, \bar{\theta}),
\end{align*}
\]
The above superfields are the generalization of the D-dimensional local fields \((B_{\mu\nu\eta}, C_{\mu\nu}, \bar{C}_{\mu\nu}, \Phi_\mu, \bar{C}_2, C_2, \bar{C}_1, C_1, \beta_\mu, \bar{\beta}_\mu)\) of the BRST and anti-BRST invariant Lagrangian density for Abelian 3-form gauge theory.

We can now expand the above superfields in terms of the D-dimensional local fields and secondary fields, e.g.;

\[
\tilde{B}_{\mu\nu\eta}(x, \theta, \bar{\theta}) = B_{\mu\nu\eta}(x) + \theta \, \bar{R}_{\mu\nu\eta}(x) + \bar{\theta} \, R_{\mu\nu\eta}(x) \\
+ i \, \theta \, \bar{\theta} \, S_{\mu\nu\eta}(x),
\]

\[
\tilde{F}_{\mu\nu}(x, \theta, \bar{\theta}) = C_{\mu\nu}(x) + \theta \, \bar{B}_1^{(1)}(x) + \bar{\theta} \, B_1^{(1)}(x) \\
+ i \, \theta \, \bar{\theta} \, S_{\mu\nu}(x),
\]

\[
\tilde{F}_{\mu\nu}(x, \theta, \bar{\theta}) = \bar{C}_{\mu\nu}(x) + \theta \, \bar{B}_2^{(2)}(x) + \bar{\theta} \, B_2^{(2)}(x) \\
+ i \, \theta \, \bar{\theta} \, \bar{S}_{\mu\nu}(x) \quad \text{etc.}
\]
Where $R_{\mu\nu\eta}(x), \tilde{R}_{\mu\nu\eta}(x), S_{\mu\nu\eta}(x), B^{(1)}_{\mu\nu}(x)$, etc., are secondary fields that are determined in the terms of local basic fields and auxiliary fields of the D-dimensional theory by exploiting the horizontality condition (HC).

**Horizontality Condition [HC] (Soul-flatness condition)**

$$\tilde{d}\tilde{B}^{(3)} = dB^{(3)}$$

(D, 2) \[\rightarrow\] (D)

Where: $d = dx^\mu \partial_\mu$, $\tilde{d} = dx^\mu \partial_\mu + d\theta \partial_\theta + d\bar{\theta} \partial_{\bar{\theta}}$

$$dB^{(3)} = \left(\frac{dx^\mu \wedge dx^\nu \wedge dx^\eta \wedge dx^\rho}{4!}\right) H_{\mu\nu\eta\rho}$$
\[ H_{\mu\nu\eta\rho} = \partial_\mu B_{\nu\eta\rho} - \partial_\nu B_{\eta\rho\mu} + \partial_\eta B_{\rho\mu\nu} - \partial_\rho B_{\mu\nu\eta} \]

: Curvature tensor remains invariant under (anti-) BRST symmetry transformations

l.h.s. has spacetime differentials as well as Grassmannian differentials

The H C condition leads to, e.g. (setting Grassmannian components = 0)

\[ R_{\mu\nu\eta} = \partial_\mu C_{\nu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu}, \]
\[ \bar{R}_{\mu\nu\eta} = \partial_\mu \bar{C}_{\nu\eta} + \partial_\nu \bar{C}_{\eta\mu} + \partial_\eta \bar{C}_{\mu\nu}, \]
\[ S_{\mu\nu\eta} = -i \left( \partial_\mu B^{(2)}_{\nu\eta} + \partial_\nu B^{(2)}_{\eta\mu} + \partial_\eta B^{(2)}_{\mu\nu} \right) \]
The insertions of the secondary fields in terms of the basic and auxiliary fields leads to the derivation of the (anti-) BRST symmetry transformations; e.g.

\[
\tilde{B}_{\mu \nu \eta}(x, \theta, \bar{\theta}) = B_{\mu \nu \eta}(x) + \theta \left[ \partial_\mu \bar{C}_{\nu \eta} + \partial_\nu \bar{C}_{\eta \mu} + \partial_\eta \bar{C}_{\mu \nu} \right] \\
+ \bar{\theta} \left[ \partial_\mu C_{\nu \eta} + \partial_\nu C_{\eta \mu} + \partial_\eta C_{\mu \nu} \right] \\
+ \theta \bar{\theta} \left[ \partial_\mu B_{\nu \eta}^{(2)} + \partial_\nu B_{\eta \mu}^{(2)} + \partial_\eta B_{\mu \nu}^{(2)} \right] \\
= B_{\mu \nu \eta}(x) + \theta(s_{ab}B_{\mu \nu \eta}) + \bar{\theta}(s_b B_{\mu \nu \eta}) \\
+ \theta \bar{\theta}(s_b s_{ab}B_{\mu \nu \eta})
\]

This implies that \( (\text{with } s_b \to \lim_{\theta \to 0} \partial_\theta, \ s_{ab} \to \lim_{\theta \to 0} \partial_\theta \ ) \)

\[
s_b B_{\mu \nu \eta} = \partial_\mu C_{\nu \eta} + \partial_\nu C_{\eta \mu} + \partial_\eta C_{\mu \nu}, \\
s_{ab} B_{\mu \nu \eta} = \partial_\mu \bar{C}_{\nu \eta} + \partial_\nu \bar{C}_{\eta \mu} + \partial_\eta \bar{C}_{\mu \nu}
\]
HC leads to the following BRST symmetry transformations

\[ s_b B_{\mu \nu \eta} = \partial_\mu C_{\nu \eta} + \partial_\nu C_{\eta \mu} + \partial_\eta C_{\mu \nu}, \quad s_b \bar{C}_{\mu \nu} = B_{\mu \nu}, \]
\[ s_b C_{\mu \nu} = \partial_\mu \beta_\nu - \partial_\nu \beta_\mu, \quad s_b \beta_\mu = \partial_\mu C_2, \quad s_b C_1 = -\bar{B}, \]
\[ s_b \bar{C}_1 = B_1, \quad s_b \bar{C}_2 = B_2, \quad s_b \bar{\beta}_\mu = F_\mu, \quad s_b \phi_\mu = f_\mu, \]
\[ s_b \bar{F}_\mu = -\partial_\mu \bar{B}, \quad s_b \bar{B}_{\mu \nu} = \partial_\mu f_\nu - \partial_\nu f_\mu, \]
\[ s_b \bar{f}_\mu = \partial_\mu B_1, \quad s_b [\bar{B}, B_1, B_2, C_2, F_\mu, f_\mu, B_{\mu \nu}] = 0 \]

The above transformations are off-shell nilpotent \( s_b^2 = 0 \)
The anti-BRST symmetry

\[ s_{ab} B_{\mu \nu \eta} = \partial_\mu \bar{C}_{\nu \eta} + \partial_\nu \bar{C}_{\eta \mu} + \partial_\eta \bar{C}_{\mu \nu}, \quad s_{ab} C_{\mu \nu} = \bar{B}_{\mu \nu}, \]
\[ s_{ab} \bar{C}_{\mu \nu} = \partial_\mu \bar{\beta}_\nu - \partial_\nu \bar{\beta}_\mu, \quad s_{ab} \bar{\beta}_\mu = \partial_\mu \bar{C}_2, \quad s_{ab} \phi_\mu = \bar{f}_\mu, \]
\[ s_{ab} C_1 = -B_1, \quad s_{ab} \bar{C}_1 = -B_2, \quad s_{ab} C_2 = \bar{B}, \]
\[ s_{ab} \beta_\mu = \bar{F}_\mu, \quad s_{ab} \bar{F}_\mu = -\partial_\mu B_2, \quad s_{ab} f_\mu = -\partial_\mu B_1, \]
\[ s_{ab} B_{\mu \nu} = \partial_\mu \bar{f}_\nu - \partial_\nu \bar{f}_\mu, \]
\[ s_{ab} [\bar{B}, B_1, B_2, \bar{C}_2, \bar{F}_\mu, \bar{f}_\mu, \bar{B}_{\mu \nu}] = 0 \]

These transformations are off-shell nilpotent \((s_{ab}^2 = 0)\)
Anticommutativity property

\[ \{ s_b, s_{ab} \} \ B_{\mu\nu\eta} \neq 0, \quad \{ s_b, s_{ab} \} \ C_{\mu\nu} \neq 0, \]
\[ \{ s_b, s_{ab} \} \ \tilde{C}_{\mu\nu} \neq 0 \]

Rest of the fields respect anticommutativity

\[ \{ s_b, s_{ab} \} \Psi = 0, \]
\[ \Psi = \beta_\mu, \bar{\beta}_\mu, f_\mu, \bar{f}_\mu, F_\mu, \bar{F}_\mu, C_1, \bar{C}_1, C_2, \bar{C}_2, \ldots \]
Superfield formalism yields following Curci-Ferrari type restriction

\[ f_{\mu} + F_{\mu} = \partial_{\mu} C_1, \quad \bar{f}_{\mu} + \bar{F}_{\mu} = \partial_{\mu} \bar{C}_1, \]

\[ B_{\mu\nu} + \bar{B}_{\mu\nu} = \partial_{\mu} \phi_{\nu} - \partial_{\nu} \phi_{\mu} \]

\[ \{s_b, s_{ab}\} B_{\mu\nu\eta} = s_b s_{ab} B_{\mu\nu\eta} + s_{ab} s_b B_{\mu\nu\eta} \]

\[ = s_b [\partial_{\mu} \bar{C}_{\mu\eta} + \partial_{\nu} \bar{C}_{\eta\mu} + \partial_{\eta} \bar{C}_{\mu\nu}] \]

\[ + s_{ab} [\partial_{\mu} C_{\mu\eta} + \partial_{\nu} C_{\eta\mu} + \partial_{\eta} C_{\mu\nu}] \]

\[ = \partial_{\mu} [B_{\nu\eta} + \bar{B}_{\nu\eta}] + \partial_{\nu} [B_{\eta\mu} + \bar{B}_{\eta\mu}] \]

\[ + \partial_{\eta} [B_{\mu\nu} + \bar{B}_{\mu\nu}] = 0 \]

on \[ B_{\mu\nu} + \bar{B}_{\mu\nu} = \partial_{\mu} \phi_{\nu} - \partial_{\nu} \phi_{\mu} \]
Similarly

\[ \{s_b, s_{ab}\} \, C_{\mu\nu} = 0, \quad \{s_b, s_{ab}\} \, \bar{C}_{\mu\nu} = 0 \]

Thus, on the constrained surface, defined by the CF-type conditions, the (anti-)BRST symmetry transformations \( s_{(a)b} \) are found to be off-shell nilpotent \( (s_{(a)b}^2 = 0) \) and absolutely anticommuting \( (s_b s_{ab} + s_{ab} s_b = 0) \)

Without knowledge of the Lagrangian density, we have derived the proper (anti-)BRST symmetry transformations
Remarks

- Off-shell nilpotency and Absolute anticommutativity
  - Superfield formalism
    (Bonora & Tonin [81, 82])

- Three CF-type conditions
  - 3-form Abelian theory

- One CF-type condition
  - 2-form Abelian theory
  \[ (B_\mu + \bar{B}_\mu = \partial_\mu \phi) \]

- One CF condition
  - 1-form non-Abelian theory
  \[ B + \bar{B} = -i (C \times \bar{C}) \]

- One CF-type condition
  - 1-form Abelian theory
  (trivial \( B + \bar{B} = 0 \))
Two CF-type conditions \[ f_\mu + F_\mu = \partial_\mu C_1, \quad \bar{f}_\mu + \bar{F}_\mu = \partial_\mu \bar{C}_1 \] Fermionic in nature

One CF-type condition Bosonic

ONLY BOSONIC \[ f_\mu + F_\mu = \partial_\mu C_1, \quad \bar{f}_\mu + \bar{F}_\mu = \partial_\mu \bar{C}_1 \]

1-form (non-)Abelian/2-form Abelian gauge theories

CF-type restriction is **ONE** of the key features of any arbitrary p-form gauge theory. Within the framework of BRST, a gauge theory is always *endowed* with CF-type restriction(s) **HALLMARK**
CF-type restrictions are (anti-)BRST invariant, e.g.

\[ s_{(a)\,b}[f_\mu + F_\mu - \partial_\mu C_1] = 0, \]
\[ s_{(a)\,b}[\bar{f}_\mu + \bar{F}_\mu - \partial_\mu \bar{C}_1] = 0, \]
\[ s_{(a)\,b}[B_{\mu\nu} + \bar{B}_{\mu\nu} - (\partial_\mu \phi_\nu - \partial_\nu \phi_\mu)] = 0 \]

This is a key consequence of our superfield formulation.
Lagrangian densities:

\[ \mathcal{L}_B = \frac{1}{24} H^{\mu\nu\eta\zeta} H_{\mu\nu\eta\zeta} + s_b s_{ab} \left[ \frac{1}{2} \bar{C}_2 C_2 - \frac{1}{2} \bar{C}_1 C_1 + \frac{1}{2} \bar{C}^{\mu\nu} C_{\mu\nu} - \bar{\beta}^\mu \beta_\mu - \frac{1}{2} \phi^\mu \phi_\mu - \frac{1}{6} B^{\mu\nu\eta} B_{\mu\nu\eta} \right] \]

\[ \mathcal{L}_{\bar{B}} = \frac{1}{24} H^{\mu\nu\eta\zeta} H_{\mu\nu\eta\zeta} - s_{ab} s_b \left[ \frac{1}{2} \bar{C}_2 C_2 - \frac{1}{2} \bar{C}_1 C_1 + \frac{1}{2} \bar{C}^{\mu\nu} C_{\mu\nu} - \bar{\beta}^\mu \beta_\mu - \frac{1}{2} \phi^\mu \phi_\mu - \frac{1}{6} B^{\mu\nu\eta} B_{\mu\nu\eta} \right] \]

where kinetic term is generated by

\[ dB^{(3)} = H^{(4)}, \quad H^{(4)} = \left( \frac{dx^\mu \wedge dx^\nu \wedge dx^\eta \wedge dx^\zeta}{4!} \right) H_{\mu\nu\eta\zeta} \]
Finally, in an explicit form, we have

\[
\mathcal{L}_B = \frac{1}{24} H^{\mu \nu \eta \zeta} H_{\mu \nu \eta \zeta} - \frac{1}{2} B^{\mu \nu} B_{\mu \nu} - B B_2 - \frac{1}{2} B_1^2 \\
+ B^{\mu \nu} \left[ \partial^\eta B_{\eta \mu \nu} + \frac{1}{2} \left( \partial_\mu \phi_{\nu} - \partial_\nu \phi_{\mu} \right) \right] + (\partial \cdot \phi) B_1 \\
+ \left( \partial_\mu \bar{C}_{\nu \eta} + \partial_\nu \bar{C}_{\eta \mu} + \partial_\eta \bar{C}_{\mu \nu} \right) \left( \partial^\mu C^{\nu \eta} \right) - (\partial \cdot \bar{\beta}) B \\
- \left( \partial_\mu \bar{\beta}_{\nu} - \partial_\nu \bar{\beta}_{\mu} \right) \partial^\mu \beta^\nu + \partial^\mu \bar{C}_2 \partial_\mu C_2 + (\partial \cdot \beta) B_2 \\
+ \left( \partial_\mu \bar{C}^{\mu \nu} + \partial^\nu \bar{C}_1 \right) f_\nu - \left( \partial_\mu C^{\mu \nu} + \partial^\nu C_1 \right) \bar{F}_\nu
\]

It should be noted that, by using the CF-conditions, the above form has been obtained.
Similarly we have

\[
\mathcal{L}_{\bar{B}} = \frac{1}{24} H^{\mu\nu\eta \zeta} H_{\mu\nu \eta \zeta} - \frac{1}{2} \bar{B}^{\mu\nu} \bar{B}_{\mu\nu} - BB_2 - \frac{1}{2} B_1^2
\]

\[
- \bar{B}^{\mu\nu} \left[ \partial^\eta B_{\eta \mu \nu} - \frac{1}{2} \left( \partial_\mu \phi_\nu - \partial_\nu \phi_\mu \right) \right] + (\partial \cdot \phi) B_1
\]

\[
+ \left( \partial_\mu \bar{C}^{\nu \eta} + \partial_\nu \bar{C}^{\eta \mu} + \partial_\eta \bar{C}^{\mu \nu} \right) \left( \partial^{\mu} C^{\nu \eta} \right) - (\partial \cdot \bar{\beta}) B
\]

\[
- \left( \partial_\mu \bar{\beta}^\nu - \partial_\nu \bar{\beta}_\mu \right) \partial^{\mu} \beta^\nu + \partial^{\mu} \bar{C}_2 \partial_\mu C_2 + (\partial \cdot \beta) B_2
\]

\[
+ \left( \partial_\mu \bar{C}^{\mu \nu} + \partial^\nu \bar{C}_1 \right) f_\nu - \left( \partial_\mu C^{\mu \nu} + \partial^\nu C_1 \right) \bar{F}_\nu
\]

The above Lagrangian densities (\(\mathcal{L}_B\) and \(\mathcal{L}_{\bar{B}}\)) are coupled but equivalent
Under the BRST and anti-BRST transformations

\[ s_b \mathcal{L}_B = \partial_\mu \left[ \left( \partial^\mu C^{\nu\eta} + \partial^\nu C^{\eta\mu} + \partial^\eta C^{\mu\nu} \right) B_{\nu\eta} + B^{\mu\nu} f_\nu \right. \\
\left. - \left( \partial^\mu \beta^\nu - \partial^\nu \beta^\mu \right) \bar{F}_\nu + B_1 f^{\mu} - B \bar{F}^{\mu} + B_2 \partial^\mu C_2 \right] \]

\[ s_{ab} \mathcal{L}_{\bar{B}} = \partial_\mu \left[ \left( \partial^\mu \bar{C}^{\nu\eta} + \partial^\nu \bar{C}^{\eta\mu} + \partial^\eta \bar{C}^{\mu\nu} \right) \bar{B}_{\nu\eta} + \bar{B}^{\mu\nu} \bar{f}_\nu \right. \\
\left. - \left( \partial^\mu \bar{\beta}^\nu - \partial^\nu \bar{\beta}^\mu \right) F_\nu + B_1 \bar{f}^{\mu} + B_2 F^{\mu} - B \partial^\mu \bar{C}_2 \right] \]

This establishes (anti-)BRST invariance.
To show the equivalence between the above Lagrangian densities and (anti-)BRST symmetries it can be checked that [LB & RPM, J. Phys. A (2010)]

\[ s_b \mathcal{L} \bar{B} = \partial_\mu [\ldots] + \text{Term that are zero on }^{\text{CF-type conditions}} \]

\[ s_{ab} \mathcal{L} B = \partial_\mu [\ldots] + \text{Term that are zero on }^{\text{CF-type conditions}} \]
Let us write one term explicitly [LB & RPM (2010)]

\[ s_b \mathcal{L}_B = -\partial_\mu \left[ (\partial^\mu C^{\nu\eta} + \partial^\nu C^{\eta\mu} + \partial_\eta C^{\mu\nu}) \bar{B}_{\nu\eta} + B^{\mu\nu} F_\nu \right. \\
- (\partial^\mu \beta^\nu - \partial^\nu \beta^\mu) \bar{f}_\nu - B_1 f^\mu + B \bar{F}^\mu - B_2 \partial^\mu C_2 \\
+ B^{\mu\nu\eta}(\partial_\nu f_\eta - \partial_\eta f_\nu) + \bar{C}^{\mu\nu} \partial_\nu B + C^{\mu\nu} \partial_\nu B_1 \left] + X \right. \\
\]

\[ X = (\partial^\mu C^{\nu\eta} + \partial^\nu C^{\eta\mu} + \partial_\eta C^{\mu\nu}) \partial_\mu [\bar{B}_{\nu\eta} + B_{\nu\eta} \\
- (\partial_\nu \phi_\eta - \partial_\eta \phi_\nu)] - [f^\mu + F^\mu - \partial^\mu C_1] (\partial_\mu B_1) \\
- [\bar{B}_{\mu\nu} + B_{\mu\nu} - (\partial_\mu \phi_\nu - \partial_\nu \phi_\mu)] (\partial^\mu f^\nu) \\
+ [\bar{f}^\mu + \bar{F}^\mu - \partial^\mu \bar{C}_1] (\partial_\mu B) + B^{\mu\nu} \partial_\mu [f_\nu + F_\nu - \partial_\nu C_1] \\
- (\partial^\mu \beta^\nu - \partial^\nu \beta^\mu) \partial_\mu [\bar{f}_\nu + \bar{F}_\nu - \partial_\nu \bar{C}_1] \]

Which is zero on the constrained surface defined by CF-conditions
Ghost Symmetries:

\[ \mathcal{L}_{(g)} = (\partial_{\mu} \bar{C}_{\nu\eta} + \partial_{\nu} \bar{C}_{\eta\mu} + \partial_{\eta} \bar{C}_{\mu\nu})(\partial^{\mu}C^{\nu\eta}) - (\partial \cdot \bar{\beta})B \\
- (\partial_{\mu} \bar{\beta}_{\nu} - \partial_{\nu} \bar{\beta}_{\mu})(\partial^{\mu}\beta^{\nu}) - BB_2 + (\partial_{\mu} \bar{C}_{\mu\nu} + \partial^{\nu} \bar{C}_{1})f_{\nu} \]

- \( \partial_{\mu}C^{\mu\nu} - \partial^{\nu}C_{1} \) \( \bar{F}_{\nu} \) + \( \partial_{\mu} \bar{C}_{2} \partial^{\mu}C_{2} + (\partial \cdot \beta)B_2 \)

The above Lagrangian density has the following symmetry transformations

\[ C_{\mu\nu} \to e^{+\Omega}C_{\mu\nu}, \quad \bar{C}_{\mu\nu} \to e^{-\Omega}\bar{C}_{\mu\nu}, \quad C_{1} \to e^{+\Omega}C_{1}, \]
\[ \bar{C}_{1} \to e^{-\Omega}\bar{C}_{1}, \quad f_{\mu} \to e^{+\Omega}f_{\mu}, \quad F_{\mu} \to e^{+\Omega}F_{\mu}, \]
\[ \bar{f}_{\mu} \to e^{-\Omega}\bar{f}_{\mu}, \quad \bar{F}_{\mu} \to e^{-\Omega}\bar{F}_{\mu}, \quad \beta_{\mu} \to e^{+2\Omega}\beta_{\mu}, \]
\[ \bar{\beta}_{\mu} \to e^{-2\Omega}\bar{\beta}_{\mu}, \quad B \to e^{+2\Omega}B, \quad B_2 \to e^{-2\Omega}B_2, \]
\[ C_{2} \to e^{+3\Omega}C_{2}, \quad \bar{C}_{2} \to e^{-3\Omega}\bar{C}_{2} \]
Conserved Charges by Noether’s Theorem:

We obtain conserved currents and they lead to the following charges

\[ Q_b = \int d^3x \left[ H^{0ijk} (\partial_i C_{jk}) + (\partial^0 C^\nu \eta + \partial^\nu C^{\eta 0} + \partial^\eta C^{0\nu}) B_{\nu \eta} + B_1 f^0 \ight. \\
- (\partial^0 \bar{\beta}^i - \partial^i \bar{\beta}^0) \partial_i C_2 - (\partial^0 \beta^i - \partial^i \beta^0) \bar{F}_i + B^{0i} f_i + B_2 \dot{C}_2 \\
- \left( \partial^0 \bar{C}^{\nu \eta} + \partial^\nu \bar{C}^\eta_0 + \partial^\eta \bar{C}^{0\nu} \right) (\partial_{\nu} \beta_\eta - \partial_{\eta} \beta_{\nu}) - B \bar{F}_0 \right] \\

\[ Q_{ab} = \int d^3x \left[ H^{0ijk} (\partial_i \bar{C}_{jk}) - (\partial^0 \bar{C}^{\nu \eta} + \partial^\nu \bar{C}^\eta_0 + \partial^\eta \bar{C}^{0\nu}) \bar{B}_{\nu \eta} + B_1 \bar{f}^0 \ight. \\
- (\partial^0 \beta^i - \partial^i \beta^0) \partial_i \bar{C}_2 - (\partial^0 \bar{\beta}^i - \partial^i \bar{\beta}^0) F_i + \bar{B}^{0i} \bar{f}_i - B \dot{\bar{C}}_2 \\
+ \left( \partial^0 C^{\nu \eta} + \partial^\nu C^{\eta 0} + \partial^\eta C^{0\nu} \right) (\partial_{\nu} \bar{\beta}_\eta - \partial_{\eta} \bar{\beta}_{\nu}) + B_2 \bar{F}_0 \right] \]
\[ Q_g \ = \ \int d^3 x \left[ 3 \dot{C}_2 C_2 - 3 \ddot{C}_2 \dot{C}_2 + (\partial^0 \bar{C}^{\nu \eta} + \partial^\nu \bar{C}^{\eta 0} + \partial^\eta \bar{C}^{0\nu}) C_{\nu \eta} - \bar{C}_1 f^0 \\
+ \ 2(\partial^0 \beta^i - \partial^i \beta^0) \bar{\beta}_i - 2(\partial^0 \bar{\beta}^i - \partial^i \bar{\beta}^0) \beta_i - \bar{C}^{0i} f_i - C^{0i} \bar{F}_i \\
+ \ 2B \bar{\beta}^0 + 2B_2 \beta^0 + C_1 \bar{F}^0 + (\partial^0 \bar{C}^{\nu \eta} + \partial^\nu \bar{C}^{\eta 0} + \partial^\eta \bar{C}^{0\nu}) \bar{C}_{\nu \eta} \right] \]

The above charges are the generators of the nilpotent and continuous (anti-)BRST symmetries and continuous ghost scale transformations

They obey the *standard BRST algebra*
The application of the continuous symmetry transformations on the above charges produces the following algebra

\[ s_b Q_b = -i \{ Q_b, Q_b \} = 0 \Rightarrow Q_b^2 = 0, \]
\[ s_{ab} Q_{ab} = -i \{ Q_{ab}, Q_{ab} \} = 0 \Rightarrow Q_{ab}^2 = 0, \]
\[ s_{ab} Q_b = -i \{ Q_b, Q_{ab} \} = 0 \Rightarrow Q_b Q_{ab} + Q_{ab} Q_b = 0, \]
\[ s_b Q_g = -i [ Q_g, Q_b ] = -Q_b \Rightarrow i [ Q_g, Q_b ] = +Q_b, \]
\[ s_{ab} Q_g = -i [ Q_g, Q_{ab} ] = +Q_{ab} \Rightarrow i [ Q_g, Q_{ab} ] = -Q_{ab}. \]
These are the standard algebra of BRST formalism. As it turns out

\[ Q_{(a)b} |phys\rangle = 0 \Rightarrow \text{First-class constraints } |phys\rangle = 0 \]

Thus, the BRST formalism gives standard results.

Superfield formulation: Any arbitrary p-form (p = 1, 2, 3, ….) Abelian gauge theory in any arbitrary D-dimensions can be described in the language of BRST approach

- Off-shell nilpotent & Absolutely Anticommuting (anti-)BRST symmetries are natural consequences!!
Abelian 1-form gauge theory

\[ A^{(1)} = dx^\mu \partial_\mu \]

\[ s_b A^{(1)} = dC^{(0)} \]

\[ s_{ab} A^{(1)} = d\bar{C}^{(0)} \]

\[ (B + \bar{B} = 0, \bar{B} = -B) \] - CF-type condition

Clustering of fields
Abelian 2-form gauge theory

\[ s_b B^{(2)} = dC^{(1)} \]
\[ s_{ab} B^{(2)} = d\bar{C}^{(1)} \]

\[ B_\mu + \bar{B}_\mu = \partial_\mu \phi \]
\[ s_b C^{(1)} = d\beta^{(0)} \]
\[ s_b B^{(3)} = dC^{(2)} \]
\[ B^{(3)} = \frac{1}{3!} (dx^\mu \wedge dx^\nu \wedge dx^\eta) B_{\mu\nu\eta} \]
\[ C^{(2)} = \frac{1}{2!} (dx^\mu \wedge dx^\nu) C_{\mu\nu} \]
Future directions:

✓ Non-Abelian Generalization

✓ Still higher p-form (p = 4, 5) theories

✓ Merging of 1-form and 3-form theories

✓ Merging of 2-form and 3-form theories
Acknowledgements:

DST, Government of India, for funding

Collaborators:

Prof. L. Bonora (SISSA, ITALY)
Mr. Saurabh Gupta (Ph. D. Student)
Mr. Rohit Kumar (Ph. D. Student)
Mr. Aradhya Shukla (Ph. D. Student)
Mr. Shri Krishna (Ph. D. Student)
Mr. Pradeep Prakash (Ph. D. Student)
Thanks