Measurement of Thermal Conductivity by Lee’s method

Aim:

To determine thermal conductivity of a bad conductor (glass) in form of a disc using Lee’s method.

Requisites:

(1) Lee’s apparatus and the experimental specimen in the form of a disc.
(2) Two thermometers, (3) Stop watch, (4) Weighing balance, (5) Special lamp stand
(6) Boiler and (7) Heater

Theory:

Thermal conductivity, \( k \), is the property of a material that indicates its ability to conduct heat. Conduction will take place if there exists a temperature gradient in a solid (or stationary fluid) medium. Energy is transferred from more energetic to less energetic molecules when neighboring molecules collide. Conductive heat flow occurs in direction of the decreasing temperature because higher temperature is associated with higher molecular energy. **Fourier’s Law** expresses conductive heat transfer as

\[
H = kA \frac{(T_2 - T_1)}{x}
\]

where \( H \) is the steady state rate of heat transfer, \( k \) is the thermal conductivity of the sample, \( A \) is the cross sectional area and \( (T_2 - T_1) \) is the temperature difference across the sample thickness ‘\( x \)’ (see Fig. 1), assuming that the heat loss from the sides of the sample is negligible. To keep the loss from the sides small, the sample is made in form of a thin disk with a large cross sectional area compared to the area exposed at the edge. Keeping ‘\( A \)’ large and ‘\( x \)’ small produces a large rate of energy transfer across the sample. Keeping \( x \) small also means that the apparatus reaches a steady state (when temperature \( T_1 \) and \( T_2 \) are constant) more quickly.

Generally speaking, there are a number of possibilities to measure thermal conductivity, each of them being suitable for a limited range of materials, depending on the thermal properties and the medium temperature. The most commonly used methods are Searle’s method and Lee’s disc method, for good and bad conductors of heat, respectively. In the experiment, we will use Lee’s disc method to determine the thermal conductivity of a bad conductor, e.g. Glass.
Description of Lee’s apparatus:

The apparatus shown in Fig. 2 consists of two parts. The lower part C is circular metal disc. The experimental specimen G, usually rubber, glass or ebonite (here it is glass) is placed on it. The diameter of G is equal to that of C and thickness is uniform throughout. A steam chamber is placed on C. The lower part of the steam chamber, B is made of a thick metal plate of the same diameter as of C. The upper part is a hollow chamber in which two side tubes are provided for inflow and outflow of steam. Two thermometers $T_1$ and $T_2$ are inserted into two holes in C and B, respectively. There are three hooks attached to C. The complete setup is suspended from a clamp stand by attaching threads to these hooks.

![Diagram of thermal conductivity measurement setup]

**Photograph of thermal conductivity measurement setup**
When steam flows for some time, the temperatures recorded \( T_1 \) and \( T_2 \) gradually remain steady. This is the steady state.

Let at the steady state, temperature of \( C = T_1 \).

Temperature of \( B = T_2 \).

Surface area of \( G = A \)

Conductivity of \( G = k \)

Thickmess of \( G = x \)

Hence amount of heat flowing through \( G \) per second, \( H \) is given by Eq. (1). When the apparatus is in steady state (temperatures \( T_1 \) and \( T_2 \) constant), the rate of heat conduction into the brass disc \( C \) is equal to the rate of heat loss from the bottom of it. The rate of heat loss can be determined by measuring how fast the disc \( C \) cools at the previous (steady state) temperature \( T_1 \) (with the top of the brass disk covered with insulation). If the mass and specific heat of the lower disc are \( m \) and \( s \), respectively and the rate of cooling at \( T_1 \) is \( dT/dt \) then the amount of heat radiated per second is,

\[
H = ms \frac{dT}{dt} \quad \text{………(2)}
\]

Equating (1) and (2) and simplifying, \( k \) can be determined as,

\[
k = \frac{ms(dT/dt)x}{A (T_2 - T_1)} \quad \text{………(3)}
\]

Procedure:

1. Fill the boiler with water to nearly half and heat it to produce steam.
2. In the mean time, take weight of \( C \) by a weighing balance. Note its specific heat from a constant table. Measure the diameter of the specimen by a scale or slide calipers, if possible. Calculate the surface area, \( A = \pi r^2 \).
3. Measure the thickness of the specimen by screw gauge. Take observations at 5 spots and take the mean value.
4. Put the specimen, steam chamber etc. in position and suspend it from the clamp stand. Insert the thermometer. Check if both of them are displaying readings at room temperature. If not, note the difference \( \theta \), is to be added to \( (T_2 - T_1) \) later.
5. Now stem is ready. Connect the boiler outlet with the inlet of the steam chamber by a rubber tube.
6. Temperatures recorded in the thermometers will show a rise and finally will be steady at \( T_1 \) and \( T_2 \).
7. Wait for 10 minutes and note the steady temperature. Stop the inflow of steam.
8. Remove the steam chamber and the specimen \( G \). \( C \) is still suspended. Heat \( C \) directly by the steam chamber till its temperature is about \( T_1 + 7^\circ \).
9. Remove the steam chamber and wait for 2 – 3 minutes so that heat is uniformly distributed over the disc C.
10. Place the insulating material on C. Start recording the temperature at ½ minute intervals. Continue till the temperature falls by $10^0$ from $T_1$.

Observations:

(I) Details of the sample G
(a) Diameter: (using scale/slide calipers)

Table-1:

<table>
<thead>
<tr>
<th>Sl No.</th>
<th>Diameter (cm)</th>
<th>Mean Diameter (cm)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td></td>
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<tr>
<td>3</td>
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</tbody>
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Surface area of G = $A = \ldots \ldots \ldots \ldots \ldots$

Thickness: (using screw gauge)

Table – 2:

Pitch = \ldots \ldots \ldots \ldots \ldots Least count=\ldots \ldots \ldots \ldots \ldots

<table>
<thead>
<tr>
<th>Sl No.</th>
<th>Initial Reading I (cm)</th>
<th>Final Reading F (cm)</th>
<th>Difference (I ~ F) in cm</th>
<th>Mean (cm)</th>
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<tr>
<td>1</td>
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(II) Details of the lower disc C
Mass of the disc, $m = \ldots \ldots$
Specific heat of the material, \( s = 380 \, \text{J/kg. °C} \)

(III) Correction of Thermometers
Room temperature recorded \( T_2 = \ldots \)
Room temperature recorded \( T_1 = \ldots \)
So correction of thermometers \( \theta = T_2 - T_1 \)

(IV) Steady Temperature
Temperature of C = \ldots
Temperature of B = \ldots
Taking thermometer error into account, the difference = \( T_2 - T_1 + \theta \)

(V) **Table-3**: Time – Temperature record during cooling

<table>
<thead>
<tr>
<th>Time (minute)</th>
<th>0</th>
<th>( \frac{1}{2} )</th>
<th>1</th>
<th>\ldots</th>
<th>\ldots</th>
<th>\ldots</th>
<th>\ldots</th>
<th>\ldots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature  ( ^\circ \text{C} )</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
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<td></td>
</tr>
</tbody>
</table>

**Graph:**

Using the data from Table – 3, plot the cooling curve (time versus temperature) and determine the slope \( \frac{dT}{dt} = \frac{\Delta T}{\Delta t} \) at the steady temperature \( T_1 \) (Fig. 3).

**Calculation:** \( k = \ldots \)

**Discussion and conclusion:**

**Probable errors and precautions:**

1. Don’t record \( T_1 \) and \( T_2 \) unless they have remained steady for at least 10 minutes.
2. The tangent to the cooling curve should be done very carefully. An error in \( \frac{dT}{dt} \) will result in a wrong result for \( k \).
3. The radiation loss only due to the surface of G in contact with C is taken into account here while there is some loss at the edges. Therefore, the diameter of the specimen disc S should be sufficiently large than its thickness.

4. Since room temperature might change during the course of the experiment, it is advised to complete the experiment quickly.

Error analysis: \( \frac{\Delta k}{k} = \sqrt{\left( \frac{\Delta (\frac{dT}{dt})}{dT} \right)^2 + \left( 2 \frac{\Delta d}{d} \right)^2 + \left( \frac{\Delta x}{x} \right)^2 + \left( \frac{\Delta T_2 + \Delta T_1}{T_2 - T_1} \right)^2} \)