

Phy 352 (Fluid Dynamics) Spring 2013, Midterm exam (30 points) You are allowed to bring 4 sheets (8 pages) of your own notes to the exam

1. (5 points) Explain why flows with very low Mach numbers can be considered quasi-hydrostatic

Ans: The Mach number represents the ratio of the inertia term (term on LHS) to the pressure gradient term (second term on the RHS).

$$\frac{d\mathbf{u}}{dt} = \mathbf{g} - \frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \nabla^2 \mathbf{u}$$

If the Mach number is very low, it means that the inertia term is unimportant, so the flow is basically hydrostatic.

2. (5 points) Consider a viscous fluid between two plane parallel plates that are separated by a distance b . Let the lower plate coincide with the x -axis. The upper plate is moving parallel to the x -axis with a speed U . All flow fields are independent of x . Show that

(a) (2.5 points) The pressure distribution is hydrostatic

(b) (2.5 points) The distribution of fluid velocity between the plates is given by $u(y) = Uy/b$

Ans: One just needs to examine the x and y components of the momentum equation (the Navier-Stokes equation). The y component of the momentum equation is $-dp/dy - \rho g = 0$; i.e., $p = p_0 - \rho g y$, which is a hydrostatic pressure distribution. The x component of the momentum equation is $\nu d^2 u/dy^2 = 0$, which gives $u = Ay + B$, where A and B are constants. The fluid is viscous; so it's at rest on the lower plate $y = 0$; which gives $B = 0$. It moves with a speed U at the upper plate, which gives $A = U/b$. Taken together, $u(y) = Uy/b$

3. (4 points)

(a) (2 points) Write down (all the elements of) the pressure tensor for a fluid with zero viscosity (i.e., an ideal fluid). Justify your answer.

Ans: All the diagonal terms are equal to $p/3$, and the off-diagonal terms are zero.

(b) (2 points) Consider a fluid with a dynamic viscosity coefficient μ which flows in the \hat{x} direction, and has a gradient of 2 s^{-1} in the \hat{y} direction. Write down all the elements of the pressure tensor.

Ans: In addition, P_{xy} (or P_{12} , if you wish) = 2μ

4. (2 point)

Consider a (small amplitude) 0.5 second sound pulse propagating adiabatically through a uniform, unbounded medium at 20 degrees celsius. The central frequency of the pulse is 200 Hz.

(a) (1 point) What is the range of frequencies present in the pulse?

Ans: The bandwidth is $1/0.5 = 2$ Hz. So the range of frequencies is 200 ± 2 Hz

(b) (1 point) What is the speed at which the pulse propagates?

Ans: Although there are many frequencies present in the pulse, it doesn't matter, for sound waves are nondispersive, and the entire pulse will propagate at around 343 m/s

5. (5 points)

One of the key steps in linearizing is neglecting the product of small quantities. For instance, in linearizing the momentum equation

$$(\rho_0 + \rho_1) \frac{\partial \mathbf{u}_1}{\partial t} + (\rho_0 + \rho_1) \mathbf{u}_1 \cdot \nabla \mathbf{u}_1 = -\nabla p_1$$

we neglect the second term on the LHS. Consider a body that measures 10 m (assume a cube) moving through a medium. The typical macroscopic timescale for the problem is 2 minutes. How large would the velocity disturbances u_1 have to be so that the linearizing assumption is **not** valid anymore?

Ans: $\mathbf{u}_1 \cdot \nabla \mathbf{u}_1 \sim O\left(u_1^2/L\right)$, and $\partial \mathbf{u}_1/\partial t \sim O\left(u_1/T\right)$. If $\mathbf{u}_1 \cdot \nabla \mathbf{u}_1 \approx \partial \mathbf{u}_1/\partial t$; in other words, $u_1 \approx L/T = 10/2 = 5$ meters/minute the linearizing assumption is not valid anymore.

6. (5 points) Consider the problem of the drag force F_D on a smooth sphere (that you are now familiar with). The quantity $F_D/(\rho U^2 L^2)$ is usually plotted as a function of the Reynolds number Re . How does $F_D/(\rho U^2 L^2)$ depend upon Re in the

(a) low Reynolds number regime (the Stokes law), and

(b) the high Reynolds number regime (turbulent flows)

Ans: In the turbulent regime, $F_D \propto U^2 L^2$, so $F_D/(\rho U^2 L^2)$ is a constant. In the low Reynolds number regime (Stoke's law), $F_D \propto U L$, so $F_D/(\rho U^2 L^2) \propto Re^{-1}$

7. (4 points) Consider an incompressible fluid flowing through a pipe. The pipe is horizontal, but its cross-sectional radius is not constant throughout its length. In a part of the pipe where its radius is 2 cm, the flow speed is 10 cm/s. What is the pressure difference between this part and a part where the pipe radius is 5 cm?

Ans: From mass conservation, $u_1 A_1 = u_2 A_2$. From Bernoulli's constant, $P_1 + (1/2)v_1^2 = P_2 + (1/2)v_2^2$. Combining, $P_1 - P_2 = (v_1^2/2)(A_1^2/A_2^2 - 1)$