Phy 352 (Fluid Dynamics) Spring 2013, Problem Set 1

1. Expand the following:
   - $a_{ij}(x_j + y_j)$
   - $(a_{ij} - a_{ji})x_i x_j$
   - Write $a_{ij}x_i y_j$ in an alternate form
   - Evaluate $a_{ij}\, x_i\, x_j$ if $a_{ij} = i - j$
2. Convince yourself that
   - $\nabla \,.\, \mathbf{u} = u_{i\,,\,i}$
   - $(\nabla \times \mathbf{u})_i = \epsilon_{i\,j\,k}\, u_{k\,,\,j}$
   - div (curl $\mathbf{u}$) $= 0$ for any vector $\mathbf{u}$
   - curl (grad ϕ) $= 0$ for any single-valued scalar ϕ
3. How should the mean free path between collisions compare with the macroscopic dimensions (say, of the room you're sitting in) in order for the fluid/continuum approximation to be valid?
4. Consider a Maxwell-Boltzmann distribution defined by
$$f(\mathbf{u}) = A \exp\left[-B(\mathbf{u} - \mathbf{u}_0)^2\right]$$
Compute the number density n (number per unit volume) by explicit integration of the distribution
$$n = \int_{-\infty}^{\infty} d^3u f(\mathbf{u})$$
Using the expression for n, show that the average velocity $\langle \mathbf{u} \rangle = \mathbf{u}_0$. What does this average velocity represent?
5. An line element along a streamline is defined by $d\mathbf{s} = (dx, dy, dz)$. Convince yourself that the definition of a streamline
$$\frac{dx}{v_x} = \frac{dy}{v_y} = \frac{dz}{v_z}$$
is equivalent to $d\mathbf{s} \times \mathbf{v} = 0$. What does this statement mean physically? (in terms of whether a fluid element can cross a streamline)
6. You will soon be introduced to the so-called "stream function", where the (2D) velocity vector of the fluid flow can be expressed in terms of a scalar stream function $\psi(x, y)$ as $\mathbf{v}(x, y) = -\nabla \times (\psi \hat{\mathbf{z}})$. Show that this description is valid only for incompressible fluids. How is this streamfunction related to streamlines?
7. Its often stated that an *irrotational* flow is one for which the net circulation is equal to 0. Why?
8. Kundu states how a flow around an object (say, around a ship) that seems (time) steady for an observer outside the object need not be so (i.e., need not seem time-steady) for an observer sitting on the object. Justify this using the concepts of Lagrangian and Eulerian derivatives.
9. Problem 4, chapter 3, Kundu & Cohen (same in the second as well as the fourth editions)