Phy 352 (Fluid Dynamics) Spring 2013, Problem Set 2

1. Consider a mass of fluid at rest that is subjected to a uniform (downward) gravitational field; i.e., the body force is
$$\mathbf{F} = -g\hat{\mathbf{z}}$$
For instance, think of a large container of water at rest in the Earth's gravitational field. Assuming that the fluid is incompressible, show that

   - The pressure increases linearly with depth from the surface
   - Assuming that the temperature throughout the fluid is constant, obtain an expression for the variation of density with vertical height. How does your answer relate to the incompressibility assumption?

2. Derive the Euler equation from
$$\frac{\partial}{\partial t}(\rho\mathbf{u}) + \nabla\,.\,(\rho\mathbf{u}\mathbf{u} + \mathbf{P}) = 0\,,$$
and the mass continuity equation, and assuming that $\mathbf{P} = p\delta_{ij}$ (what does this mean, and why is this condition taken to be a proxy for an inviscid fluid?).

3. What is the physical interpretation of the ij-th component of the pressure tensor?

4. Consider the vorticity field $\omega = \nabla \times \mathbf{u}$ of any field. Can this vorticity field have sources or sinks? How does this relate to Kelvin's vorticity theorem?

5. We have worked out the problem of water flowing out of an orifice on the side of a tank using the Bernoulli constant. This was done assuming that the height of the water column does not change with time. Suppose you now want to figure out how long it will take for the tank to empty. Clearly, you cannot do it with this assumption. Relax this assumption (i.e., assume that the height can change with time) and figure out how long it will take for the tank to empty.

6. Consider water flowing through a pipe of varying cross-section. Intuitively, you know that the fluid speeds up when it enters the narrower portion of the pipe. Justify this.

7. A jet of water with a diameter 8 cm and a speed of 25 m/s is incident normally on a large plate. Find the force needed to hold the plate stationary. Note the adjective "large"; how would your answer change if the plate were not large; i.e., if its dimensions were comparable with that of the jet cross-section?

8. We have worked out the (2D) velocity distribution for potential (i.e., inviscid, irrotational, incompressible) flow around a smooth sphere.

   (a) Where in the flow is the maximum speed attained, and how much is this value?

   (b) Use Bernoulli's constant to figure out the distribution of pressure around the sphere. Does this tell you anything about the absence of a drag force?

9. Consider a viscous liquid flowing along a (very long) cylinder. Show that the variation of the axial flow speed along the cross-section of the cylinder is a centered parabola; the axial flow speed is maximum along the cylinder axis and it falls to zero at the cylinder walls in a parabolic fashion.