

Statistical Thermodynamics (CHM422-2014)

Assignment 2

Probability Problems

1. In a certain colony, 60% of the families own a car, 30% own a house and 20% own both a car and house. If a family is randomly chosen, what is the probability that this family owns a car or house, but not both?

2. A blood test is 95% effective in detecting a certain disease, when it is in fact present. The test also yields false positive result for 1% of the healthy persons tested. If 0.5% of the population actually have the disease, what is the probability that a person has a disease given that his test result is positive ?

3. Let A_1, A_2, \dots, A_n be independent events. Show that the probability that none of the events A_1, A_2, \dots, A_n occur is less than $\exp[-\sum_{i=1}^n P(A_i)]$.

4. A fair die is rolled repeatedly. Compute the probability of event A having a 2 will show up before a 5.

5. If $P(A^c)=0.3$, $P(B)=0.4$ and $P(AB^c) = 0.5$, find $P(B|AB^c)$.

6. Suppose that the distribution function of X is given by F(b).

$$F(b) = \begin{cases} 0 & \text{for } b < 0 \\ \frac{b}{4} & \text{for } 0 \leq b < 1 \\ \frac{1}{2} + \frac{b-1}{4} & \text{for } 1 \leq b < 2 \\ \frac{11}{12} & \text{for } 2 \leq b < 3 \\ 1 & \text{for } 3 \leq b \end{cases}$$

Find $P(X=i)$, $i=1,2,3$ (b) Find $P\{1/2 < x < 3/2\}$

7. A normal six-sided die has been loaded so that the probability of rolling a 6 is twice as much as that of rolling any other number. What is the probability (a) of rolling a 6 with this die? (b) of rolling a 4? (c) How much more likely are you to roll two of these loaded dice and have the sum of the two dice be 8 than you are with the roll of two normal dice?

8. As discussed in the class, say there are 40 H atoms and 10 D atoms in a box. How many possible molecular distributions can be created? What is the number of HD for the most probable distribution? How probable is the most probable distribution? (only

indicate how you will calculate it?) What is the relative probability of For 35 H_2 , 15HD , and 5 D_2 molecules?

Chemical Distribution

9. Chlorine naturally occurs with two major isotopes ^{35}Cl and ^{37}Cl . On earth, approximately 76% of all chlorine atoms are ^{35}Cl and 24% are ^{37}Cl . Given that 1.5 mol of chlorine atoms in their natural abundances, what is the most probable distribution of $^{35}\text{Cl}_2$, $^{37}\text{Cl}_2$, $^{35}\text{Cl}^{37}\text{Cl}$?

10. Starting with the previous problem, what is the new most probable distribution if (a) 0.5 mol of $^{35}\text{Cl}_2$ is added? (b) if 0.25 mol of $^{35}\text{Cl}^{37}\text{Cl}$ is added? (c) if 0.5 mol of $^{37}\text{Cl}_2$ and 0.25 mol of $^{35}\text{Cl}^{37}\text{Cl}$ are added?

11. The three natural isotopes of oxygen and their natural abundance are ^{16}O (99.757%), and ^{17}O (0.038%), and ^{18}O (0.205%). What is the most probable distribution of O_2 molecules among the six different possibilities in a sample containing 2 mol of natural O_2 .