Reasoning*

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Include other rules of inference:
If P then Q, P therefore Q
If P then Q, not Q, therefore not P
If P then Q, If Q then R, therefore If P then R
If P then either Q or R, P and not R, therefore Q.

1 Introduction to Reasoning

1.1 What is reasoning?

If we see dark clouds in the sky, we infer that it is likely to rain. If we hear the sound of laughter behind a closed door, we infer that behind that door there is either a person or a device like a tape recorder or TV set. The experience of seeing the dark clouds and of hearing the sound of laughter is an instance of the ***grounds*** that lead us to the ***conclusion*** about the likelihood of rain and about a person or device behind the door. The process of ***inference*** connects the grounds to the conclusion.

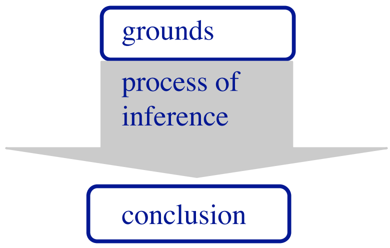

figure 1

In some cases, the grounds of inference and the steps leading from the grounds to the conclusion can be expressed (either verbally or using some other symbolic system) such that someone else can scrutinize the process and make an assessment its quality. For instance, given the information that Zeno is a spider, we can infer that Zeno has eight legs. If we are asked to express it verbally, we may do so as follows:

> Zeno is a spider.
> Therefore, it is reasonable to conclude that Zeno has eight legs.

If we are asked how exactly the conclusion of Zeno having eight legs follows from Zeno being a spider, we might flesh out our reasoning as follows:

> Zeno is a spider.
> All spiders have eight legs.
> Therefore, it is reasonable to conclude that Zeno has eight legs.

Likewise, given the information that Leda's room is 10 ft wide and 15 ft long, it is reasonable to conclude that the area of the room is 150 sq ft. We can articulate the process of inference in a combination of words and numbers as follows:

> Width of Leda's room = 10 ft
> Length of Leda's room = 15 ft
> Hence, the area of Leda's room = 150 sq.ft

Reasoning is a form of inference that can be expressed in words or other symbols such that the legitimacy of the process can be evaluated if needed.

Implicit in what we have said above is the idea that not all forms of inference can be overtly expressed. Suppose you walk into an auditorium with a friend, look around at the seated people, and tell your friend, "There seems to almost a hundred people here." You didn't actually count the number of people and arrive at this conclusion, instead, you made in inference, based on your visual experience. If your friend challenged you to express the steps of your inference in words or numbers, you may not be in a position to do so.

Reasoning is an integral component of the various aspects of academic inquiry, including

i) looking for answers/solutions to questions/problems (methodology);
ii) justifying/proving our answers/solutions (justification); and
iii) evaluating the merit of the answers/solutions (critical thinking).

Given the centrality of reasoning in inquiry, it is important for us to understand the nature of reasoning in academic inquiry. In what follows, we will explore forms of reasoning that can be expressed in words (qualitative reasoning) or in numbers (quantitative reasoning).

1.2 Premises and conclusions

When formulated as statements expressed in words or numbers, the grounds that we appeal to in our inferences are called ***premises***. Take a look at the premises in our examples of Zeno's legs and area of Leda's room:

(**1**)	Zeno is a spider.	Premise 1
	All spiders have eight legs.	Premise 2
	Therefore, it is reasonable to conclude that Zeno has eight legs.	Conclusion

Likewise, given the information that Leda's room is 10 ft wide and 15 ft long, it is reasonable to conclude that the area of the room is 150 sq ft. We can articulate the process of inference in a combination of words and numbers as follows:

(**2**)	The width of Leda's room = 10 ft	Premise 1
	The length of Leda's room = 15 ft	Premise 2
	Hence, the area of Leda's room = 150 sq.ft	Conclusion

Take another example to illustrate the concept of premises. Suppose you are told that all gleeps are dovines, all dovines have green eyes, and Blimpsey is a gleep. You are now asked: "What is the colour of Blimpsey's eyes?" Even though you have no idea what gleeps and dovines are, and who Blimpsey is, you will have no trouble inferring that Blimpsey's eyes are green. The reasoning in this example can be expressed as:

(**3**)	All gleeps are dovines.	Premise 1
	All dovines have green eyes.	Premise 2
	Blimpsey is a gleep.	Premise 3
	Therefore, Blimpsey's eyes are green.	Conclusion

We may now see reasoning as *the process of arriving at a conclusion from a given set of premises:*

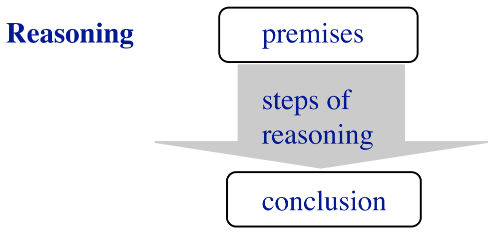

figure 2

1.3 Spelling out the chain of reasoning

Example (3) leaves out an intermediate step in reasoning. Given that all gleeps are dovines, and that Blimpsey is a gleep, we conclude that Blimpsey is a dovine. Given that all dovines have green eyes and that Blimpsey is a dovine, we can conclude that Blimpsey has green eyes. Here is a version of (2) that spells out the implicit steps:

(**4**)

All gleeps are dovines.	Premise 1
Blimpsey is a gleep.	Ppremise 3
Therefore: Blimpsey is a dovine.	Conclusion (intermediate)

Blimpsey is a dovine.	
All dovines have green eyes.	Premise 2
Therefore: Blimpsey has green eyes.	Conclusion (final)

1.4 Implicit and explicit premises

Intuitively, we recognize the reasoning in example (2) as quite legitimate. However, there are two crucial premise that are taken for granted in this reasoning, namely, that Leda's room is a rectangle, and the area of a rectangle is the product of its length and width. To be explicit, therefore, we would spell out these implicit premises as follows:

(**5**)

Leda's room is a rectangle.	premise 1
The length of Leda's room = 15 ft.	premise 2
The width of Leda's room = 10 ft.	premise 3
The area of a rectangle = length x breadth.	premise 4
Therefore: The area of Leda's room = 150 sq ft.	conclusion

In the actual practice of reasoning, we often leave out premises that are taken for granted by the writer and the reader. This happens even in the most rigorous forms of reasoning such as that involved in mathematical proofs.

Though the inexplicitness illustrated of missing steps and implicit premises is not a serious flaw in the particular examples given above, we should watch out against the danger of the inexplicit premises being false or untenable, and implicit steps involving an invalid derivation. For instance, even though most rooms are rectangular in shape, there is a possibility that Leda's room is elliptical. If so, the conclusion in (2)/(5) would be false. When we evaluate a sample of reasoning, therefore, it is important to unearth the hidden premises and check if any of them is false or untenable.

2 Typology of Reasoning

2.1 Modus Ponens

2.1.1 Classical Modus Ponens

To repeat, reasoning is the process of arriving at inferences from a given body of information. The discipline of logic is the systematization of reasoning. It studies the abstract patterns of good reasoning and systematizes them, such that we can:

- further develop and strengthen our reasoning capacity, and
- evaluate the legitimacy of the reasoning that we are exposed to.

To do this, logic has to articulate explicit principles of good reasoning, so that we can distinguish between good reasoning and bad reasoning. To this end, logicians formulate both the information and the inferences involved in reasoning as ***propositions***. The propositions carrying the information are the premises, and the propositions carrying the inferences that we derive from the premises are the conclusions.

Information *Premises*	steps of reasoning ⟶	Inferences *Conclusions*

How do we distinguish between good and bad reasoning? Take the following examples:

Example A	Example B
All humans are mammals.	All humans are mammals.
Zenta is a human.	Xilo is a mammal.
Therefore Zenta is a mammal.	Therefore Xilo is a human.

Intuitively, we recognize A to be an instance of good reasoning and B to be an instance of bad reasoning. We therefore expect logic to tell us why this is so.

To unearth what is bad about B, the first step is to express the propositions in such a way that their logical structure becomes clear. The proposition expressed by the sentence *All humans are mammals* is the same as "For any x, if x is a human, x is a mammal." Let us restate examples A and B as A' and B':

Example A'	Example B'
For any x, if x is a human, x is a mammal.	For any x, if x is a human, x is a mammal.
Zenta is a human.	Xilo is a mammal.
Therefore Zenta is a mammal.	Therefore Xilo is a human.

Let us denote *x is a human* as P and *x is a mammal* as Q:

For any x, if $\underbrace{\text{x is a human}}_{P}$, $\underbrace{\text{x is a mammal}}_{Q}$.

We now see that the sentence *All humans are mammals* expresses a complex proposition consisting of two simple propositions P and Q, and has the form: If P, then Q.

Logicians use an arrow to denote the "if…then" relation, and write "If P, then Q" as: P → Q. We can now express the logical structure of examples A' and B' as A" and B":

	Example A"	Example B"
Premise 1:	$P \rightarrow Q$	$P \rightarrow Q$
Premise 2:	P	Q
Conclusion:	Therefore Q	Therefore P

Why is the structure in B" bad, in contrast to that in A"? An answer calls for the concept of ***rules of inference***, the *abstract patterns that sanction a conclusion from a given set of premises*. One such rule of inference in deductive logic is called *Modus Ponens*, which says:[1]

Rule of inference: ***(Classical) Modus Ponens***

Given that:	*If P is true, then Q is true*
and	*P is true*
it is reasonable to conclude that:	*Q is true.*

Modus Ponens sanctions the reasoning in A", but not in B". The reasoning in B" is bad because the rule of Modus Ponens does not sanction it, nor have we postulated any other rule of inference that legitimizes it. (We will postulate more rules of inference as we proceed.)

1 Why we call it *Classical* Modus Ponens will become clear in the next section.

The assumption that we are making above is that *each step in a chain of reasoning from premises to conclusions must be mediated by a legitimate rule of inference*:

Premises

<u>Reasoning</u> $\downarrow$ Rules of inference

Conclusion

2.1.2 Classical vs. Probabilistic Modus Ponens

Consider the following examples of reasoning:

(**6**) Every human being is mortal.
Socrates was a human being.
Therefore, Socrates was mortal.

(**7**) All human beings have their heart on the left side and their liver on the right side.
Plato was a human being.
Therefore, Plato had the heart on the leftt side and the liver on the right side.

The reasoning in these examples appeals to the rule of Modus Ponens discussed in the previous section. Example (6) is found in most introductory books on logic. We will refer to this as ***classical modus ponens***.

Classical Modus Ponens has the following property:

If the premises are true, and
the rule is applied correctly,
the conclusion must necessarily be true.

It cannot be the case that the statements that every human being is mortal and that Socrates was a human being are both true, and yet it was not true that Socrates was mortal. Thus:

*If the premises are **totally certain**, and the application of the rule is correct, Classical Modus Ponens yields a **totally certain** conclusion.*

Now, we haven't seen any evidence to believe that the statement that every human being is mortal is incorrect: we have not come across any immortal human being yet. However, the first premise in (7) is not accurate. There is a rare genetic condition in healthy human beings called *situs inversus* in which the heart is on the right and the liver is on the left. The frequency of *situs inversus* varies across populations, but apparently it is less than 1 out of 10, 000. But the point is that if you take a large enough sample, you might come across a human being with heart on the right and liver on the left. Hence, the reasoning in (7) should be revised as (8).

(**8**) 99.99% of human beings have the heart on the left side and the liver on the right side.
Plato was a human being.
Therefore, the probability of Plato having the heart on the left side and the liver on the right side is 0.9999.

As it happens, there are also people with the heart on the left, and yet the liver is also on the left. This condition is called *situs inversus incompletes*. It appears to be the case that 1 out of 22,000 cases of *situs inversus* are instances of *situs inversus incompletes*. Thus one out of 220, 000, 000 human beings have both their heart and liver on the left side of their body:

(**9**) 1 out of 220,000,000 human beings have both the heart and the liver on the left side of their body.
Plato was a human being.
Therefore, the probability of Plato having both the heart and the liver on the left side of their body is approximately 0.000000045

The reasoning illustrated in (8) and (9) is probabilistic. To see the distinction between (7) and (8) more clearly, let us re-formulate (7) in numerical terms.

(**7'**) 100% of human beings have the heart on the left side and the liver on the right side.
Plato was a human being.
Therefore, the probability of Plato having the heart on the right side and the liver on the right side is 1.

(7') is an instance of classical deduction in which truth is either 1 (true) or 0 (false), i.e., conclusions are totally certain. (8) and (9) are examples of probabilistic reasoning that allow degrees of certainty, truth and falsity ranging from 1 to 0.

What would a Probabilistic Modus Ponens look like? To answer this question, let us re-formulate Classical Modus Ponens in quantitative terms (i.e., in terms of numbers):

(Classical) Modus Ponens (in quantitative terms)

Given that:	*If P is true, then the probability of Q is 1.*
and	*P is true*
it is reasonable to conclude that:	*the probability of Q is 1.*

It is now easy to see that the probabilistic version of Modus Ponens can be formulated as follows, replacing 1 with *n*, where n ranges from 1 to 0.

Probabilistic Modus Ponens (in quantitative terms)

Given that:	*If P is true, then the probability of Q is n.*
and	*P is true*
it is reasonable to conclude that:	*the probability of Q is n.*

Probabilities are expressed qualitatively in ordinary language in terms of adjectives like *likely*, *most likely*, *almost certainly*, and so on. So the qualitative version of the Probabilistic Modus Ponens would be:

Probabilistic Modus Ponens (in qualitative terms)

Given that:	*If P is true, then Q is (most) likely to be true.*
and	*P is true*
it is reasonable to conclude that:	*Q is (most) likely to be true.*

2.1.3 Classical vs. Defeasible Modus Ponens

Suppose we are told that Xena goes to her office every weekday, and that 2 January 2007 was a weekday. We will conclude that Xena went to her office on 2 Jan. 2007.

Now let us suppose that we are given the additional information that Xena does not go to her office when she is sick, and that she was sick on 2 Jan. 2007. We would most probably conclude that Xena did not go to her office on 2 Jan. 2007, overriding the earlier conclusion that she went to her office on that day.

Now we are told that there was an emergency in Xena's office on 2 Jan. 2007, and whenever there is an emergency, she goes to her office. Did she go to her office on 2 Jan. 2007? Our answer would probably depend upon whether the emergency was strong enough to override the sickness factor.

The shifts in the conclusions in this example illustrates an important property of a large class of human reasoning, namely: w*hen additional information becomes available, a conclusion that was accepted earlier as correct can be rejected as incorrect.*

A mode of reasoning that has the above property is called ***defeasible reasoning***. Classical Modus Ponens is non-defeasible. To see this clearly, let us derive the conclusions in the example of Xena and her office using Classical Modus Ponens.

(**10**) a. If x is a weekday, Xena goes to her office on day x.
b. If Xena is sick on day x, she does not go to her office
c. 2 Jan. 2007 was a weekday.

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d. Therefore, by (a) and (c), Xena went to her office on 2 Jan. 2007.

(**11**) a. If x is a weekday, Xena goes to her office on day x.
b. If Xena is sick on day x, she does not go to her office
c. 2 Jan. 2007 was a weekday.
d. Xena was sick on 2 Jan. 2007.

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e. By (a) and (c), Xena went to work on 2 Jan. 2007.
f. By (b) and (d), Xena did not go to her office on 2 Jan. 2007.
g. By (e) and (f), therefore, Xena went to her office and did not go to her office on 2 Jan. 2007.

The reasoning in (11), using Classical Modus Ponens, yields a **logical contradiction**. This is because in this mode of reasoning, there is no way to cancel an earlier inference as long as the premises remain. To model the mode of reasoning that blocks the conclusion that Xena went to her office on Jan. 2, we need a rule of inference that is capable of suppressing an otherwise valid conclusion. This is provided by Defeasible Modus Ponens:

Defeasible Modus Ponens

Given	(i) *if P is true, then Q is true*
	(ii) *if R is true then not-Q is true*
	(iii) *P is true.*
	(iv) *R is true*
and	(v) (i) is stronger than (ii)
it is reasonable to conclude that	*Q is true and not-Q is false.*

Thus, (v) cancels the inference from the combination of (ii) and (iv), namely, that Q is not true. This is the crucial aspect of Defeasible Modus Ponens. For this mode to work, we need to survey all the candidates that are potential conclusions, check if there is a logical contradiction, and if there is, cancel the weaker conclusion:

(**12**) a. If x is a weekday, Xena goes to her office on day x.
b. If Xena is sick on day x, she does not go to her office
c. (b) is stronger than (a).
d. 2 Jan. 2007 was a weekday.

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e. Candidate by (a) and (d): Xena went to her office on 2 Jan. 2007.
f. No rival candidate, so we conclude that Xena went to her office on 2 Jan. 2007.

(**13**) a. If x is a weekday, Xena goes to her office on day x.
b. If Xena is sick on day x, she does not go to her office
c. (b) is stronger than (a).
d. 2 Jan. 2007 was a week day.
e. Xena was sick on 2 Jan. 2007.

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f. Candidate: by (a) and (d), Xena went to work on 2 Jan. 2007.
g. Candidate: by (b) and (e), Xena did not go to her office on 2 Jan. 2007.
h. By (c), candidate (g) wins, and hence Xena did not go to her office on 2 Jan. 2007.

Let us take another example. If we are told that Nixon is a Republican and Republicans are not pacifists, we conclude by Classical Modus Ponens that Nixon is not a pacifist. If we are told that

Nixon is a Quaker, and that Quakers are pacifists, Classical Modus Ponens yields the conclusion that Nixon is a pacifist. If we are told that Nixon is both a Quaker and Republican, classical deductive reasoning leads to a conclusion that is logically contradictory, namely, that Nixon is not a pacifist and Nixon is a pacifist.

(**14**)	Nixon is a Quaker.	Nixon is a Republican.
	Quakers are pacifists.	Republicans are not pacifists.
	∴ Nixon is a pacifist.	∴ Nixon is not a pacifist.

Nixon is a pacifist and Nixon is not a pacifist.

In a system of defeasible reasoning, otherwise valid conclusions from one set of premises can be defeated by stronger conclusions from other premises. If we are told that Nixon is both a Quaker and Republican, we have to decide which of these conflicting pulls is stronger. If being a Republican has priority over being a Quaker, for instance, we conclude that Nixon is not a pacifist in a system of defeasible reasoning:

(**15**)	Nixon is a Quaker.	Nixon is a Republican.
	Quakers are pacifists.	Republicans are not pacifists.
	∴ Nixon is a pacifist.	∴ Nixon is not a pacifist.

Nixon is not a pacifist.

Moral reasoning, that is, reasoning from moral principles to moral judgments, crucially needs Defeasible Modus Ponens. Suppose we accept the following principles in our moral theory.

A. i. If an action brings about the death of a human being, given an option between action and inaction, choosing action is immoral.

ii. If an action saves the life of a human being, given an option between action and inaction, choosing inaction is immoral.

B. i. If an action causes suffering to a human being or increases the suffering, given an option between action and inaction, choosing action is immoral.

ii. If an action prevents or alleviates the suffering of a human being, given an option between action and inaction, choosing inaction is immoral.

C. i. Given an option between telling a lie and not telling a lie, choosing to tell a lie is immoral.

ii. Given an option between speaking the truth and not speaking the truth, choosing not to speak the truth is immoral.

Take the following scenario borrowed from philosopher Peter Singer as an illustration of moral dilemmas. Imagine that you are a Christian living in Germany during Hitler's time. In your basement is a Jewish family hiding from the Nazis. One day, the Nazi secret police knock on your door, and ask if you are harboring any Jews in your house. What is the right thing to do, tell the truth and send the Jewish family to a prison camp, or tell a lie and save the family?

Most of us would opt for the latter. Given the moral theory that includes A-C above with the specification that A and B are stronger than C, the reasoning that underlies this conclusion can be given as follows.

(**16**) a. If an action brings about the death of a human being, given an option between action and inaction, choosing action is immoral. (Ai)

b. If an action causes suffering to a human being or increases the suffering, given an option between action and inaction, choosing action is immoral. (Bi)

c. Given an option between telling a lie and not telling a lie, choosing to tell a lie is immoral. (Ci)

d. (Ai) and (Bi) are stronger than (Ci)

e. If we tell the Gestapo about the Jews hidden in our house, it would cause suffering and death for them.

f. If we tell the Gestapo that there are no Jews hiding in our house, we are telling a lie.

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g. Candidate: by (a), (b) and (e), it is immoral to tell the Gestapo that there are Jews hiding in our house.

h. Candidate: by (c) and (f), it is immoral to tell the Gestapo that there are no Jews hiding in our house.

i. By (d), the winner is (g). Hence, we conclude that it is immoral to tell the Gestapo that there are Jews hiding in our house.

2.2 Reverse Modus Ponens

2.2.1 Abduction

Compare the reasoning in (17a) and (17b):

(**17**) a. Whenever it rains, the streets get wet. It was raining last night. Therefore, the streets must have got wet.

b. Whenever it rains, the streets get wet. The streets are wet now. Therefore it must have rained.

(17a) is an example of the familiar Modus Ponens. Given that (i) rain causes wetness of the streets, and (ii) rain (the cause) is observed, we conclude that wetness of the streets (the effect) must have happened. In other words, we infer the effect from the cause in (17a).

In contrast, we are inferring the cause from the effect in (17b), reversing the direction of inference. It was philosopher Charles Sanders Peirce who, noticing the use of this form of reasoning in everyday life and academic knowledge, called it ***abduction*** (to distinguish this mode of reasoning from ***deduction***.)

Abductive reasoning is common in the justification of interpretations, in almost all academic fields outside of mathematics and logic. The form of reasoning that doctors use in diagnosing diseases from symptoms, for instance, is abductive. Take the following example:

(**18**) When someone has a heart attack, (s)he tends to have a feeling of strangulation, pain in the chest radiating to the left shoulder and arm, abnormal perspiration, shortness of breath, and nausea.

Fanny Jenkins has just experienced a feeling of strangulation, pain in the chest radiating to the left shoulder and arm, abnormal perspiration, shortness of breath, and nausea.

Therefore it is reasonable to conclude that Fanny Jenkins had a heart attack.

Similarly, a great deal of the reasoning used in a law court is also abductive.[2]

[2] For instance, consider this. If someone shattered a windowpane from outside in order to break into the apartment, there would be broken glass inside near the window. In addition to the door being wide open and the missing stereo, I see broken glass inside the apartment near the window. It is therefore reasonable to conclude that someone broke into the house by shattering the windowpane.

It is important distinguish valid abduction from the form of illegitimate reasoning illustrated by examples such as those in (19):

(**19**) a. Whenever Leda teases Zeno, Zeno cries.
Zeno is crying now. Therefore, Leda must have teased him.

b. All horses are mammals. A rabbit is a mammal. Therefore a rabbit is a horse.

The conclusions in (19a) and (19b) are not derivable through Modus Ponens. In other words, the reasoning in (19a) and (19b) are not validated through Modus Ponens. But that by itself is not sufficient to explain why they are examples of bad reasoning, because Modus Ponens does not validate any of the examples of good reasoning in (17b) and (18) either.

To see why (17b) and (18) are examples of good reasoning while (19a) and (19b) are not, it is necessary to spell out the rule of abduction:

Abduction

Given that:		*P is the best available explanation for Q*
and		*Q is true*
it is reasonable to conclude that:		*P is true*
in the absence of	(i) evidence to the contrary, and	
	(ii) a better or equally good alternative explanation for *Q*.	

The crucial ingredient here is: *in the absence of evidence to the contrary and a better or equally good alternative explanation for Q*. This specification is absent in Modus Ponens.

In the case of the streets being wet, we have not found any counterevidence (evidence to show that it did not rain), and in the case of the doctors observations, we have not found any obvious alternative causes. Hence the reasoning in both cases is legitimate. In the case of Zeno crying, however, we are aware of other potential causes that could have led to the effect of Zeno crying: he could have fallen down and hurt himself, Mary could have scolded him, his teacher may have given him a poor grade, and so on. Likewise, there are many animals other than rabbits that are mammals, and hence the rule of abduction does not legitimize (19b).

When it rains, the streets get wet.
The streets are wet.
Therefore, it must have rained. (valid abduction, alternative causes unavailable)

When Leda beats Zeno, Zeno cries.
Zeno is crying now.
Therefore, Leda must have beaten Zeno. (invalid abduction, alternative causes available)

We accepted the conclusion that it must have rained because we couldn't think of a better or equally good explanation. Suppose someone proposes the following explanation:

When the water truck goes down the streets watering the ground, the streets get wet.
The streets are wet.

Therefore, the water truck must have gone down the street.

We now have a competing explanation, and our previous conclusion is no longer valid. We must therefore look for further evidence to choose between the two. Suppose we examine the sidewalk and the lawns near by, and find that these are not wet. The water truck hypothesis explains why only the streets are wet, while the rain hypothesis incorrectly predicts that the sidewalk and the lawns also would be wet. Hence, we choose the water truck hypothesis. If, on the other hand, we find that the sidewalk and the lawns are also wet, we choose the rain hypothesis.

Which hypothesis would you choose if you found that the lawns are wet but not the sidewalk? Why? We leave that for you to gnaw on.

2.2.2 Speculative deduction

Consider the following example of reasoning:

(**20**) During a total solar eclipse, the positions of stars near the sun as observed from the earth appear farther away from the sun than their positions under normal conditions. This puzzling phenomenon can be explained by assuming that the gravitational pull of the sun causes the light from the stars to bend, as predicted by Einstein's theory of gravity. Since no other theory explains the observations, we accept Einstein's theory as correct.

This is an instance of ***speculative deductive*** reasoning, a special type of reasoning that is characteristic of the justification of scientific theories. Given an observation (e.g., The streets are wet) and a "theory" that we take to be correct (e.g., Whenever it rains, the streets get wet.), abductive reasoning provides justification for *an interpretation (explanation/analysis) of the observation within the theory*. Given a set of observations, speculative deductive reasoning provides justification for *the theory itself on the basis of the observations*.

Given below is another example of speculative deductive reasoning to support the theory of continental drift in geography proposed by Alfred Wegener in 1912:

(**21**) The shape of the east coast of South America fits neatly with the shape of the west coast of South Africa. Fossils of mesosaurus (small freshwater reptiles) are found only in two different parts of the earth, namely, Brazil (South America) and Africa. Dinosaur fossils are scattered in identical rock strata, again in the east coast of Brazil and west coast of Africa. We can provide an explanation for the above facts if we assume that South America and West Africa formed a single landmass at an earlier point in time, and drifted apart subsequently (continental drift hypothesis). In the absence of a better or equally good alternative explanation for the above observations, we accept the continental drift hypothesis as correct.

Our last example is a speculative deductive argument to establish gravity and the law of gravitation:

(**22**) Suppose we assume that
(a) there is such a thing as gravity, and
(b) it is governed by a law that gravitational attraction between any two bodies is proportional to the product of their masses and indirectly proportional to the square of the distance between them.
This law of gravity, combined with other laws, correctly explains the generalizations of falling bodies on the earth as well as the generalizations of planetary motion. In the absence of contrary evidence, and of an alternative theory, it is reasonable to conclude that (a) and (b) are correct.

Notice that both abductive reasoning and speculative deductive reasoning are forms of *evidence-based* reasoning: given what we experience or observe as the grounds (evidence), these forms of reasoning allow us to justify a conclusion as an explanation for our experiences/observations. As pointed out earlier, Charles Sanders Peirce who was responsible for pointing attention to the distinction between abduction and deduction didn't distinguish between abduction and speculative deduction: he viewed both as "inference to the best explanation".[3]

[3] Speculative deductive reasoning is used to justify scientific theories. The grounds for justification in such justification are the data. In contrast, abductive reasoning is used to justify an interpretation within a theory. The grounds in such justification are not only the data, but also the theory itself.

Notice also that the rule of abduction that we gave earlier equally fits speculative deduction. If we wish to distinguish between the two in terms of a rule of inference, we would need to specify that in abduction, the relevant explanatory principle we are appealing to is already established in the particular domain, while in speculative deduction, the principle is a newly proposed one. The rule for speculative deduction is given below, along with the revised rule of abduction:

Abduction (revised)

Given that:	*Theory T is true*
	P is the best available explanation for Q within T
and	*Q is true*
it is reasonable to conclude that :	*P is true*
in the absence of (i) evidence contrary to P, and	
(ii) a better/equally good alternative explanation for Q within T.	

Speculative Deduction

Given that:	*Theory T is the best available explanation for Q*
and	*Q is true*
it is reasonable to conclude that :	*T is true*
in the absence of (i) evidence contrary to T, and	
(ii) a better/equally good alternative explanation for Q.	

2.3 Reasoning from Sample to Population

Most introductory textbooks on logic make a distinction between deductive and inductive reasoning. The example in (5a) reproduced below is an example of *deductive* reasoning, in contrast to (23), which is regarded as an example of *inductive* reasoning.

(**5**) a. Zeno is a spider. — premise 1
All spiders have eight legs. — premise 2

Therefore: Zeno has eight legs. — conclusion

(**23**) All the spiders that we have observed so far have eight legs. — premise

Therefore, in the absence of evidence to the contrary,
I conclude that all spiders have eight legs. — conclusion

To see what is going on in the contrast between (5a) and (23), let us go through a thought experiment. Imagine that you are shown a barrel of balls, and are told that all the balls in the barrel are red. You are now instructed to close your eyes, and pick a hundred balls randomly from the barrel, and are asked what colour the balls are. Your answer would be: red. Given that the colour of the ***population*** of the balls in the barrel is red, it follows logically that the colour of the balls in the ***sample*** drawn from the barrel is also red.

(**24**) Population to sample (P-to-S)

All the balls in this barrel are red. — population

↓

Therefore, the 100 balls in the sample that I have picked from this barrel are red. — sample

The reasoning you have used in this case is that of deduction, the relevant rule being that of Classical Modus Ponens.[4]

[4] *Classical Deductive Reasoning* uses the rule of Classical Modus Ponens. Those who teach logic in the Departments of philosophy and mathematics would probably object, but we may say that *Probabilistic Deductive Reasoning* uses

Now consider a different scenario. You are shown a barrel of balls, but are not told the colour of the balls. You are instructed to pick a hundred balls randomly from the barrel. You see that the colour of all the balls in the sample is red. You are now asked what colour the balls in the barrel are. Your answer would be that they are red: given that all the balls in the sample drawn from the barrel are red, it is most likely – though not completely certain – that the colour of the population of balls in the barrel is red.

(**25**) <u>Sample to population (S-to-P)</u>

The 100 balls in the sample that I have picked from this barrel are red. — sample

↓

Therefore, in the absence of evidence to the contrary, I conclude that all the balls in this barrel are red. — population

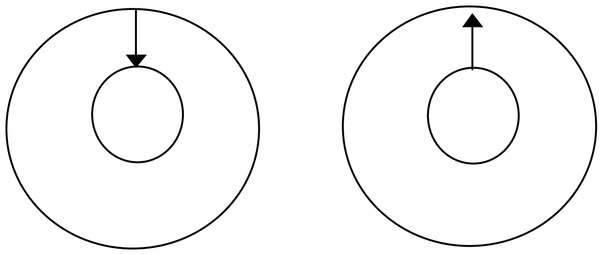
Most logic textbooks use the term *induction* to refer to the mode of reasoning illustrated in (25), to contrast it with the *deduction* in (24).

Given the properties of a population (a set), P-to-S reasoning allows us to infer the properties of a particular sample (a proper subset) of the population. Given the properties of a sample of a population, S-to-P reasoning allows us to infer the properties the population:

<u>P-to-S reasoning</u>	<u>S-to-P reasoning</u>
Population	Sample
↓	↓
Sample	Population

In terms of Venn diagrams, the distinction between the two would be as follows:

<u>P-to-S reasoning</u> <u>S-to-P reasoning</u>



We could also say that P-to-S reasoning allows us to infer the particular from the general, while S-to-P reasoning allows us to infer the general from the particular.

<u>P-to-S reasoning</u>	<u>S-to-P reasoning</u>
General	Particular
↓	↓
Particular	General

As in the case of reasoning involving Probabilistic Modus Ponens, Defeasible Modus Ponens, and Reverse Modus Ponens, the sample to population reasoning illustrated here lacks total certainty. Even if you take a sample of a thousand balls and discover all of them to be red, there is still a chance that one of the balls in the barrel is white, and that you happened to not notice it. In contrast, its deductive counterpart is totally certain: if it is true that all the balls in the barrel are red, it cannot be the case that the balls in the sample are anything but red.

Probabilistic Modus Ponens, and *Defeasible Deductive Reasoning* employs Defeasible Modus Ponens. The terminology is unimportant: what is important for us is the use of rules of inference appropriate to each context, and an understanding of contrasts such as probabilistic vs. non-probabilistic, defeasible vs. non-defeasible, quantitative vs. qualitative, population-to-sample vs. sample-to-population, and so on.

To see this more sharply, let us go back to the example of *situs inversus incompletes*. We know that one in 220,000,000 human beings who have their heart on the left have their liver also on the left. If you take two or three random samples of a thousand human beings each, chances are that you will observe that every human who has the heart on the left has the liver on the right. If you conclude on the basis of this evidence that every human being who has the heart on the left has the liver on the right, you would be wrong. The risk of being wrong is unavoidable in this mode of reasoning.

Until recently, the role model for reasoning in academia was the kind of Classical Deductive Reasoning that the Greek mathematician Euclid introduced in his proofs for theorems, codified by philosopher Aristotle. Inductive reasoning, lacking the certainty of deductive reasoning, was somehow seen as inferior.[5]

Notice that in (24), if the premise is true, it cannot be the case that the conclusion is false. In contrast, the conclusion in (25) (though reasonable) can be false even if the premise is true: there may be one white ball somewhere in the barrel that might not have been included in the sample. This possibility of error is acknowledged by the phrase *in the absence of evidence to the contrary*. One way to play it safe is to modify the conclusion to reflect the uncertainty, as in (26):

(**26**) All the balls in the sample of 1000 balls from this population are red.
Therefore, it is reasonable to conclude that all the 10,000 balls in this population are most likely to be red.

The Non-probabilistic Sample-to-Population Reasoning in (25) involves the risk of the conclusion being false. In contrast, the Probabilistic Sample-to-Population Reasoning in (26) is cautious. From a different perspective, we could say that (25), unlike (26), employs "defeasible" reasoning: when contrary evidence is discovered, the conclusion is "defeated" and hence abandoned (or modified).

As in the case of Modus Ponens discussed earlier, S-to-P reasoning can be quantitative or qualitative. Compare the quantitative reasoning in (27a) with qualitative reasoning in (27b) and (27c):

(**27**) a. 995 Martians in our representative random sample of 1,000 Martians have purple eyes. Therefore, it is reasonable to conclude that 99.5% of Martians have purple eyes.

b. Every Martian in our large representative random sample of Martians has purple eyes. Therefore, it is reasonable to conclude that all Martians are likely to have purple eyes.

c. Every Martian in our representative sample of Martians has purple eyes. Therefore, until we find Martians with non-purple eyes, it is reasonable to conclude that all Martians have purple eyes.

(27a) illustrates quantitative probabilistic S-to-P reasoning (statistical reasoning). (27b) is its qualitative probabilistic counterpart. (24c) illustrates qualitative non-probabilistic S-to-P reasoning.

The rules of probabilistic non-defeasible and non-probabilistic defeasible S-to-P reasoning can be stated as follows:

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[5] Though the distinction between (24) and (25) is quite clear, an accurate definition of deductive and inductive reasoning is far from trivial. One reason is that the term "inductive" has been used for a rag-bag of different kinds of reasoning, equating "inductive" with everything that is outside of classical deductive, covering not only the Sample-to-Population reasoning, but also what we have called Probabilistic Deduction, Defeasible Deduction, Abduction, and Speculative Deduction. In this use, the concept of "inductive reasoning" is not particularly useful.

Sample to population: probabilistic non-defeasible

Given that:	*Q is true of a large representative random sample of X*
it is reasonable to conclude that	*Q* is likely to be true of X.

Sample to population: non-probabilistic defeasible

Given that:	*Q is true of a sample of X*
it is reasonable to conclude that	*Q is true of X*
in the absence of evidence to the contrary.	

Both P-to-S and S-to-P reasoning are used in various fields of academic inquiry. In university courses, quantitative probabilistic P-to-S reasoning is typically taught under probability theory in mathematics. Its qualitative counterpart is taught as probability logic in philosophy. Quantitative probabilistic S-to-P reasoning is taught as statistics in mathematics, while its qualitative counterpart is taught under the broad rubric of inductive reasoning in philosophy.

2.4 Analogical Reasoning

No survey of the modes of reasoning would be complete without mention of analogical reasoning. Let us begin with an example. The English government has a law protecting animals in experiments, requiring that animals belonging to a privileged class that feels pain should not be operated on without anesthesia. Now, pain is an internal experience that cannot be directly observed. How do we determine whether or not a given species of animal is able to feeling pain?

How do each of us know that other human beings are capable of experiencing pain? The strategy is as follows. I know that under certain kinds of external stimuli (e.g., wounds, falls, burns) I feel pain, and when I feel pain, I tend to behave in certain ways (e.g., tears, certain facial expressions, body postures, noises). When other human beings behave in parallel ways in response to parallel stimuli, I infer that they have the parallel internal experience of pain. This inference is corroborated by what they say about their internal experience. It is therefore justifiable to conclude that human beings are capable of feeling pain.

We extend the same strategy to animals, except that they cannot report their pain to us. If we find that an animal responds to pain-inducing stimuli in ways parallel to the pain response of human beings, we conclude that the animal is experiencing pain. The type of reasoning that allows us to make such inferences is analogical reasoning.

Analogical reasoning involves making an inference about X based on (a) the analogy (equivalence) between X and Y, and (b) what we know about Y. When we observe a dog wounded in a traffic accident, for instance, we infer that the dog must be in pain, based on the analogy between humans and dogs and what we know about humans. The structure of this piece of reasoning can be unpacked as follows:

(28) Dogs are analogous to humans. When humans are wounded, they experience pain. Therefore, it is reasonable to conclude that when dogs are wounded, they experience pain.

Given below are additional examples of analogical reasoning:

(29) a. The human heart has a structure and a function. Its structure is broadly that of a complex piston. What it does is to pump the blood from the heart to the other parts of the body. Now, we all agree that we cannot really understand the structure of the heart without understanding its function. Likewise, we cannot understand the structure of language unless we understand what it is used for.

b. We know that waves in water exhibit the property of interference. If we assume that light travels in a vacuum analogous to waves in a medium, it would follow that light would also exhibit the property of interference. This prediction is borne out. Hence it is reasonable to conclude that light travels as waves.

The reasoning in (29b) involves the analogy of light and waves, and hence is an example of speculative deductive analogical reasoning. The example in (29a) involves analogical but not speculative deductive reasoning. The central rule that sanctions analogical inferences are given below:

Rule of analogical reasoning

Given that:	*x is analogous to y*
and	*P is true of y*
it is reasonable conclude that:	*P is true of x*

To illustrate:

Given that:	Dogs are analogous to humans.
and	Humans experience pain when wounded.
it is reasonable to conclude that:	Dogs experience pain when wounded.

Recall that the conclusions in many of the modes of reasoning discussed earlier need caveats like "in the absence of evidence to the contrary" and "in the absence of a better or equally good alternative". Such caveats are characteristic of defeasible reasoning, those in which the conclusion can be defeated by further evidence or by a better or equally good explanation. Defeasible S-to-P reasoning, abductive reasoning, and speculative-deductive reasoning have the property of defeasibility. So does analogical reasoning.

Exercises

1. Given below are additional examples of reasoning that contain implicit premises. Make the reasoning explicit by supplying the missing premise(s) in each case. Check whether the premises you have unearthed are acceptable.

a. The murder weapon has Zeno's fingerprints on it.
Therefore, Zeno is the murderer.

b. Leda drinks whisky every day.
Therefore, Leda is immoral.

c. Dino is a prostitute.
Therefore Dino is immoral.

d. Blimpsey flunked all her exams.
Therefore Blimpsey is not intelligent.

e. On a scale of 1 to 10, Zeno has a score of 9.8 in his student feedback.
Therefore Zeno is an excellent teacher.

2. Convert the following statements into the "If… then…" format illustrated in section 2.1.1:

a. All slaidins are tall.
b. No mammal has wings.
c. Only birds have wings.

3. Represent the information in (2a) – (2c) in terms of Venn diagrams.

4. Consider the following promise from a teacher:

Anyone who wears shoes in this class will get an A for the assignment on reasoning.

You are told that Bill wears shoes but Susan doesn't. Meena has an A, but Jacob has a B. Your task is to determine whether the teacher's promise has been kept with respect to these students. You can interview the students to elicit the relevant information, but are allowed to ask each of them only one question. Also, you have to select as few students as possible.

Which students would you interview to check the truth of the proposition? What question would you ask each one?

5. Consider the following scenarios:

> You are asked to make an estimate of the density of trees (number of trees per square kilometer) in North America.
>
> You are asked to estimate the proportion of red and white balls in a barrel containing ten thousand balls (without actually counting all the red and white balls in the barrel.)

What additional precautions do you need to take for the first task?

6. An argument is *valid* if its conclusion follows from its grounds.[6] The argument below is valid in one mode of reasoning but invalid in another. What are the two modes of reasoning?

> Wherever there is fire, there is smoke. There is smoke coming from that apartment. Therefore, in the absence of evidence to the contrary and an alternative explanation, it is reasonable to conclude that there is fire in that apartment.

7. Specify the modes of reasoning you would use in the justification of each of these claims:

a. There is a strong correlation between IQ scores language competence.
b. A forensic specialist's conclusion that the cause of death was strangling, on the basis of the evidence of bruise marks around the neck.
c. The conclusion that the distinction between objects and adjuncts is legitimate, on the basis of evidence from passives, agreement, case marking, and topicalization.
d. Jane's three month old baby boy couldn't possibly have told her that he loved Emily Dickinson's poetry.

[6] Those who have taken an introductory course in philosophy would notice that our definition of validity deviates from the definition given in logic textbooks. You will see at a later point why we choose not to adopt the textbook definition.