Phy 352 (Fluid Dynamics) Spring 2013, Midterm exam (30 points) You are allowed to bring 4 sheets (8 pages) of your own notes to the exam

1. (5 points) Explain why flows with very low Mach numbers can be considered quasi-hydrostatic

   Ans: The Mach number represents the ratio of the inertia term (term on LHS) to the pressure gradient term (second term on the RHS).
   $$\frac{d\mathbf{u}}{dt} = \mathbf{g} - \frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2 \mathbf{u}$$
   If the Mach number is very low, it means that the inertia term is unimportant, so the flow is basically hydrostatic.

2. (5 points) Consider a viscous fluid between two plane parallel plates that are separated by a distance b. Let the lower plate coincide with the x-axis. The upper plate is moving parallel to the x-axis with a speed U. All flow fields are independent of x. Show that

   (a) (2.5 points) The pressure distribution is hydrostatic

   (b) (2.5 points) The distribution of fluid velocity between the plates is given by $u(y) = Uy/b$

   Ans: One just needs to examine the x and y components of the momentum equation (the Navier-Stokes equation). The y component of the momentum equation is $-dp/dy - \rho g = 0$; i.e., $p = p_0 - \rho g y$, which is a hyrdostatic pressure distribution. The x component of the momentum equation is $\nu d^2u/dy^2 = 0$, which gives $u = Ay + B$, where A and B are constants. The fluid is viscous; so its at rest on the lower plate $y = 0$; which gives $B = 0$. It moves with a speed U at the upper plate, which gives $A = U/b$. Taken together, $u(y) = Uy/b$

3. (4 points)

   (a) (2 points) Write down (all the elements of) the pressure tensor for a fluid with zero viscosity (i.e., an ideal fluid). Justify your answer.

   Ans: All the diagonal terms are equal to $p/3$, and the off-diagonal terms are zero.

   (b) (2 points) Consider a fluid with a dynamic viscosity coefficient μ which flows in the $\hat{\mathbf{x}}$ direction, and has a gradient of 2 s^{-1} in the $\hat{\mathbf{y}}$ direction. Write down all the elements of the pressure tensor.

   Ans: In addition, P_{xy} (or P_{12}, if you wish) $= 2\mu$

4. (2 point)

   Consider a (small amplitude) 0.5 second sound pulse propagating adiabatically through a uniform, unbounded medium at 20 degrees celsius. The central frequency of the pulse is 200 Hz.

   (a) (1 point) What is the range of frequencies present in the pulse?

   Ans: The bandwidth is $1/0.5 = 2$ Hz. So the range of frequencies is 200 ± 2 Hz

   (b) (1 point) What is the speed at which the pulse propagates?

   Ans: Although there are many frequencies present in the pulse, it doesn't matter, for sound waves are nondispersive, and the entire pulse with propagate at around 343 m/s

5. (5 points)

   One of the key steps in linearizing is neglecting the product of small quantities. For instance, in linearizing the momentum equation
   $$(\rho_0 + \rho_1)\,\frac{\partial \mathbf{u}_1}{\partial t} + (\rho_0 + \rho_1)\mathbf{u}_1 \,.\, \nabla\, \mathbf{u}_1 = -\nabla\, p_1$$
   we neglect the second term on the LHS. Consider a body that measures 10 m (assume a cube) moving through a medium. The typical macroscopic timescale for the problem is 2 minutes. How large would the velocity disturbances u_1 have to be so that the linearizing assumption is **not** valid anymore?

   Ans: $\mathbf{u}_1 \,.\, \nabla\, \mathbf{u}_1 \sim O\left(u_1^2/L\right)$, and $\partial \mathbf{u}_1/\partial t \sim O\left(u_1/T\right)$. If $\mathbf{u}_1 \,.\, \nabla\, \mathbf{u}_1 \approx \partial \mathbf{u}_1/\partial t$; in other words, $u_1 \approx L/T = 10/2 = 5$ meters/minute the linearizing assumption is not valid anymore.

6. (5 points) Consider the problem of the drag force F_D on a smooth sphere (that you are now familiar with). The quantity $F_D/(\rho\, U^2\, L^2)$ is usually plotted as a function of the Reynolds number Re. How does $F_D/(\rho\, U^2\, L^2)$ depend upon Re in the

   (a) low Reynolds number regime (the Stokes law), and

   (b) the high Reynolds number regime (turbulent flows)

   Ans: In the turbulent regime, $F_D \propto U^2\, L^2$, so $F_D/(\rho\, U^2\, L^2)$ is a constant. In the low Reynolds number regime (Stoke's law), $F_D \propto U\, L$, so $F_D/(\rho\, U^2\, L^2) \propto Re^{-1}$

7. (4 points) Consider an incompressible fluid flowing through a pipe. The pipe is horizontal, but its cross-sectional radius is not constant throughout its length. In a part of the pipe where its radius is 2 cm, the flow speed is 10 cm/s. What is the pressure difference between this part and a part where the pipe radius is 5 cm?

   Ans: From mass conservation, $u_1\, A_1 = u_2\, A_2$. From Bernoulli's constant, $P_1 + (1/2)v_1^2 = P_2 + (1/2)v_2^2$. Combining, $P_1 - P_2 = (v_1^2/2)(A_1^2/A_2^2 - 1)$