Phy 352 (Fluid Dynamics) Spring 2011, Quiz 1

1. (3 points) An observer on a river bank observes that the fluid flow around a ship moving in the river is time-steady (i.e., doesn't change with time as far as (s)he can tell). Would an observer on the ship also conclude that the fluid flow around the ship is time-steady?

   Ans: No. For the observer on the river bank (and Eulerian observer), a steady flow means that $\partial/\partial t$ (of any quantity, like the velocity) = 0. On the other, the relevant quantity for observer on the ship is d/dt, which will not be zero even if $\partial/\partial t$ is zero. There is the advective term $\mathbf{u} \,.\, \nabla$ that is clearly non-zero.

2. (4 points) A city water tower is x metres high. What is the maximum pressure in water pipes that are buried y metres underground? (2 points) What is the maximum velocity of water that is sprayed from a hose that is situated z metres above the ground? ($z < x$) (2 points)

   Ans: Maximum pressure under the ground (from Bernoulli's constant, with velocity set to zero) = $\rho\, g\, (x + y)$. (Atmospheric pressure acts on all the parts). To get the maximum speed of water from the hose, we set $(1/2)v^2 = g(x - z)$. Don't be misled by the extra depth beneath the ground!

3. (3 points) You observe that a ball of radius a that is falling through a uniform fluid (of infnite extent) with density ρ under the influence of gravity attains a constant terminal speed after some time. You also observe that the terminal speed of a ball with radius 9 cm is 9 times as large as that on a ball with radius 3 cm. Both the balls are made of the same material. What kind of drag force are the balls subjected to:

   (a) $F_{\mathrm{D}} \propto \rho\, a\, \mu\, U$, or
   (b) $F_{\mathrm{D}} \propto \rho\, a^2\, U^2$?

   Ans: If the ball has attained a constant terminal speed, it means that its in dynamical equilibrium; the net force on it is zero.

$$F_{\mathrm{gravity}} = \frac{4}{3}\, \pi\, a^3\, \rho\, g = F_{\mathrm{D}}$$

   The terminal speed is observed to be $\propto a^2$; the only way this can happen is if $F_{\mathrm{D}} \propto \rho\, a\, \mu\, U$. With the other drag law, $U \propto \sqrt{a}$.

4. (5 points) Consider an inviscid fluid at rest (in a very large container) under the influence of a constant gravitational acceleration g throughout its volume.

   (a) (2 points) If the density throughout the fluid is assumed to be constant, how does the pressure inside the fluid vary with height from the bottom of the container?
   (b) (3 points) If, on the other hand, the temperature throughout the fluid is constant (the density need not be constant). How does the density of the fluid vary with height from the bottom of the container?

   Ans: Start with the Euler equation. No velocities, since the fluid is at rest, and the only relevant dimension is the height z, so

$$\frac{1}{\rho}\, \frac{dP}{dz} = g$$

   If ρ is constant, this shows that the pressure decreases linearly with z from the bottom of the container. If, on the other hand, we want to relate the pressure to the temperature, $P = \rho\, k\, T$. Substituting in the equation above, and since temperature is constant (but the density need not be so),

$$k\, T\, \frac{1}{\rho}\, \frac{d\rho}{dz} = g$$

This predicts that the density will decrease exponentially with height from the bottom of the container (gravitational stratification). Think about the *scale height* of the stratification; i.e., the e-folding lengthscale for the density stratification.

1 Useful information

1.1 Relation between the Lagrangian and Eulerian derivatives

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + (\mathbf{u} \,.\, \nabla)$$

1.2 The Euler equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \,.\, \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \mathbf{F} \tag{1}$$

1.3 The Navier-Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \,.\, \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \mathbf{F} + \nu \nabla^2 \mathbf{v} \tag{2}$$

1.4 The Bernoulli constant

$$\frac{1}{2}\,v^2 + \frac{P}{\rho} + gh = \text{constant} \tag{3}$$