Phy 352 (Fluid Dynamics) Spring 2013, Quiz 2 (15 points) Closed book, closed notes

1. (5 points) For a normal shock in an ideal fluid, what is the downstream Mach number ($\mathcal{M}_2$) for an infinitely strong shock?

   Ans: From the usual shock jump conditions, one can derive

$$\mathcal{M}_2^2 = \frac{\mathcal{M}_1^2 + 2/(\gamma - 1)}{(2\gamma/(\gamma - 1))\mathcal{M}_1^2 - 1},$$

   where $\mathcal{M}_1$ denotes the upstream Mach number (usually referred to as the "Mach number of the shock"). When $\mathcal{M}_1 \to \infty$,

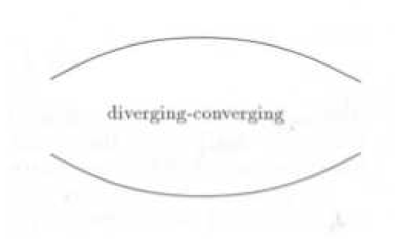
$$\mathcal{M}_2^2 = \frac{\gamma - 1}{2\gamma}$$

   For an isothermal shock, $\mathcal{M}_2 \to \infty$.

2. (5 points) You are operating a household mixer to churn some fruit juice. Assume the mixer jar to be cylindrical, with a radius of 20 cm. You estimate that the mixer is consuming some 100 watts per kg. What is the characteristic speed of the largest eddies? If the smallet eddies are around 2 cm in size, how fast are they moving?

   Ans: $\epsilon \sim u_{\rm large}^3/l_{\rm large} = u_{\rm small}^3/l_{\rm small} = 100$ W/kg. $l_{\rm large} = 20$ cm, $l_{\rm small} = 2$ cm. So $u_{\rm large} \approx 271$ cm/s, $u_{\rm small} \approx 125$ cm/s

3. (5 points) Consider a diverging-converging pipe (as shown in the figure). What kinds of flows do you think are possible? e.g., subsonic to supersonic through a sonic point, subsonic everywhere, without a sonic point, but with a minimum in the middle, etc. You need to provide concrete justifications for each of your answers.



   Ans: In the "middle", where $dA/dx = 0$, one can have either a sonic point ($\mathcal{M} = 1$) or an extremum in du/dx. Consider a flow that starts out subsonic on the left: it decelerates, so it can never pass through a sonic point in the middle. But it can reach a velocity minimum and start accelerating afterwards. *So one can have a flow that is subsonic everywhere, with a minimum in the middle.* Of course, it could eventually turn supersonic far to the right, but that's another matter. If the flow starts out supersonic on the left, it accelerates, so it will only become more supersonic, and will never pass through a sonic point. But it will start decelerating after the midpoint. *So one can have a flow that is supersonic everywhere, with a maximum in the middle.*

1 Useful information

1.1 The Euler equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \,.\, \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \mathbf{F} \tag{1}$$

1.2 The Navier-Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \,.\, \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \mathbf{F} + \nu\nabla^2\mathbf{v} \tag{2}$$

1.3 The Bernoulli constant

$$\frac{1}{2}\,v^2 + \frac{P}{\rho} + gh = \text{constant} \tag{3}$$

1.4 Conserved quantities

Mass conservation:

$$\rho\,A\,u = \text{Constant}$$

Momentum conservation:

$$p + \rho\,u^2 = \text{Constant}$$

Energy conservation:

$$\frac{1}{2}\,u^2 + \frac{\gamma}{\gamma - 1}\,\frac{p}{\rho} = \text{Constant}$$

For flow in a channel,

$$(\mathcal{M}^2 - 1)\,\frac{1}{u}\,\frac{du}{dx} = \frac{1}{A}\,\frac{dA}{dx}$$